

Compiler Design

Compiler: A high level overview

Compilers: \rightarrow TOC \rightarrow Grammar (CFG, RG), RE

pre req: \rightarrow TOC

\rightarrow Basic ^{working} knowledge of C/Java

{ Variables, keywords, Identifiers }

GNU C-Compiler

gcc

TurboC (IDE)

Borland C

Visual C++

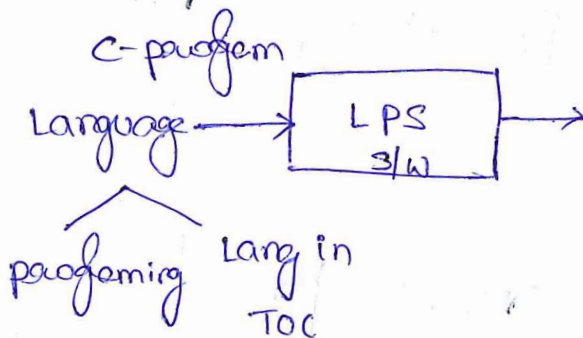
Java Compiler

i.c \rightarrow gcc \rightarrow object code
?

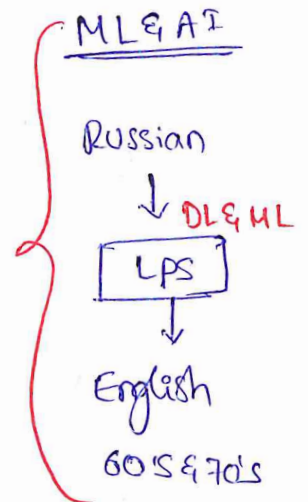
\$ gcc i.c

\$./a.out

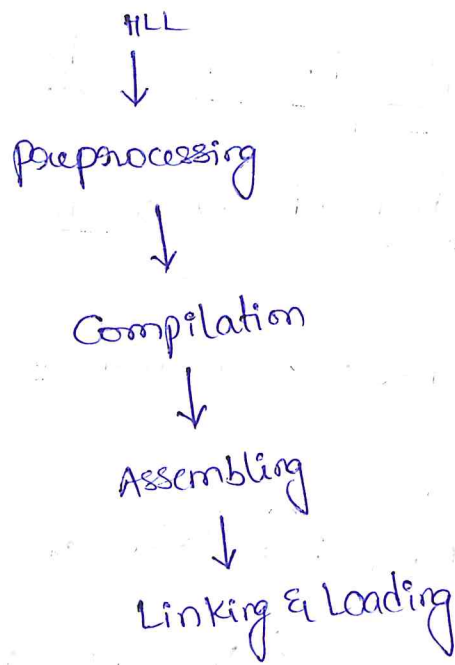
Language processing System: (LPS)



obj-code
another
Language



②



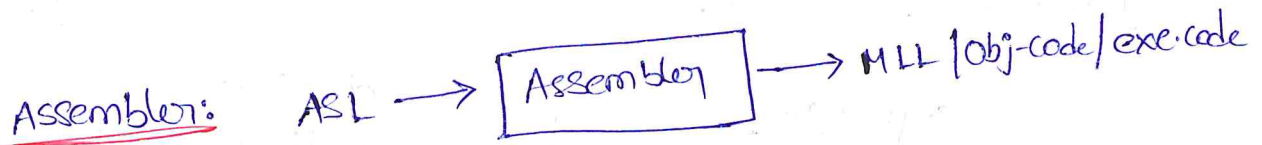
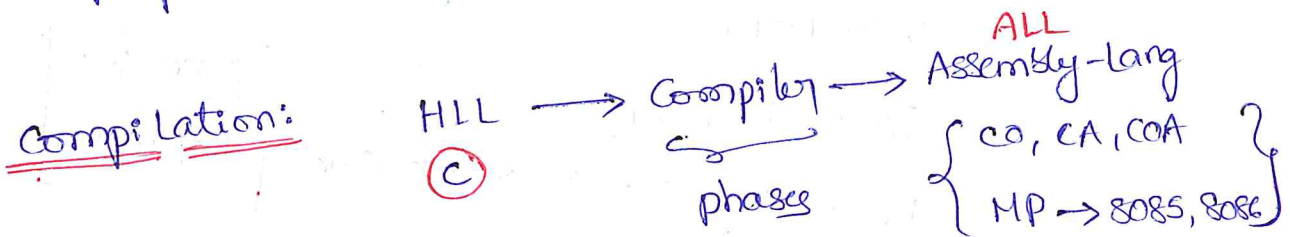
preprocessing:

```

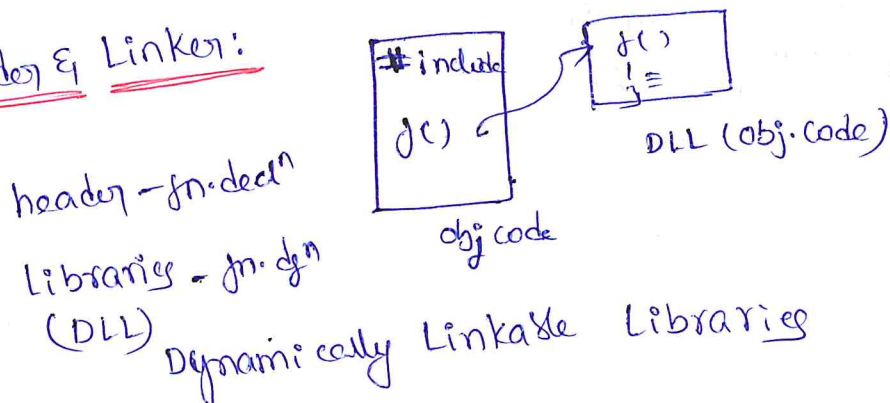
  # include <stdio.h>
  # include <stdlib.h>
  # define PI 3.1415
  
```

Header file
Macro

preprocessor handles all header files and macros.

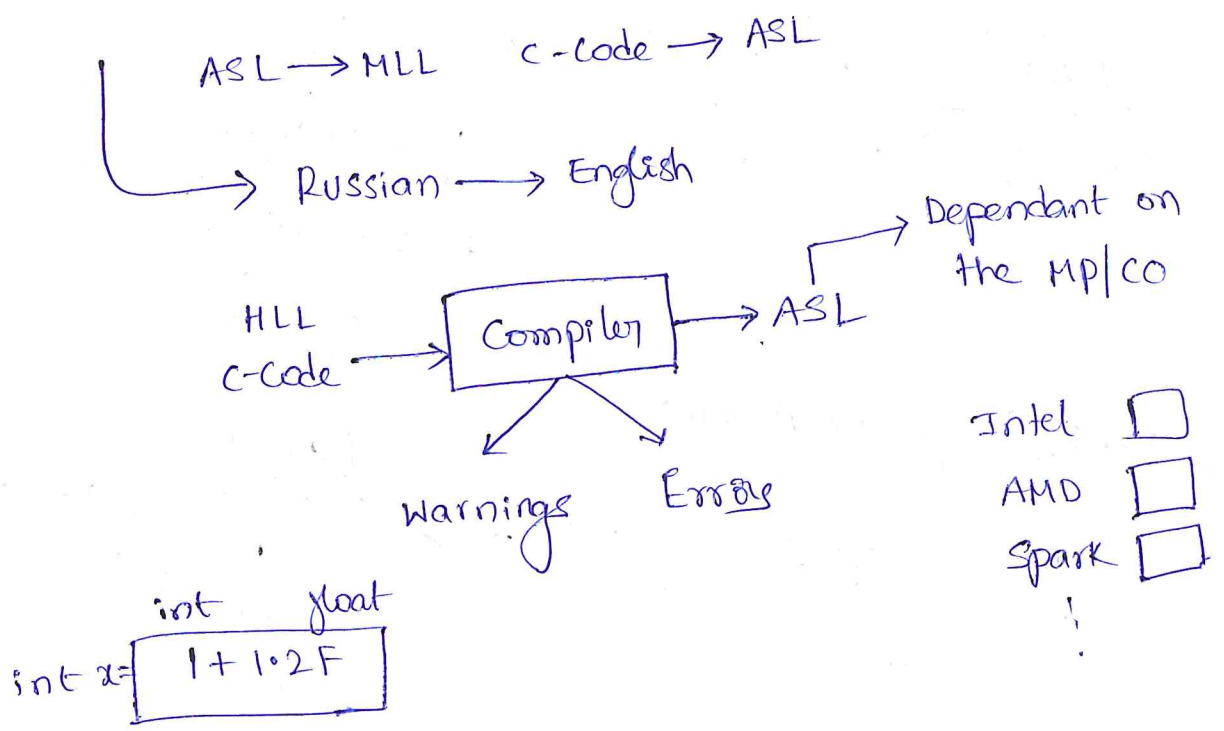


Loader & Linker:

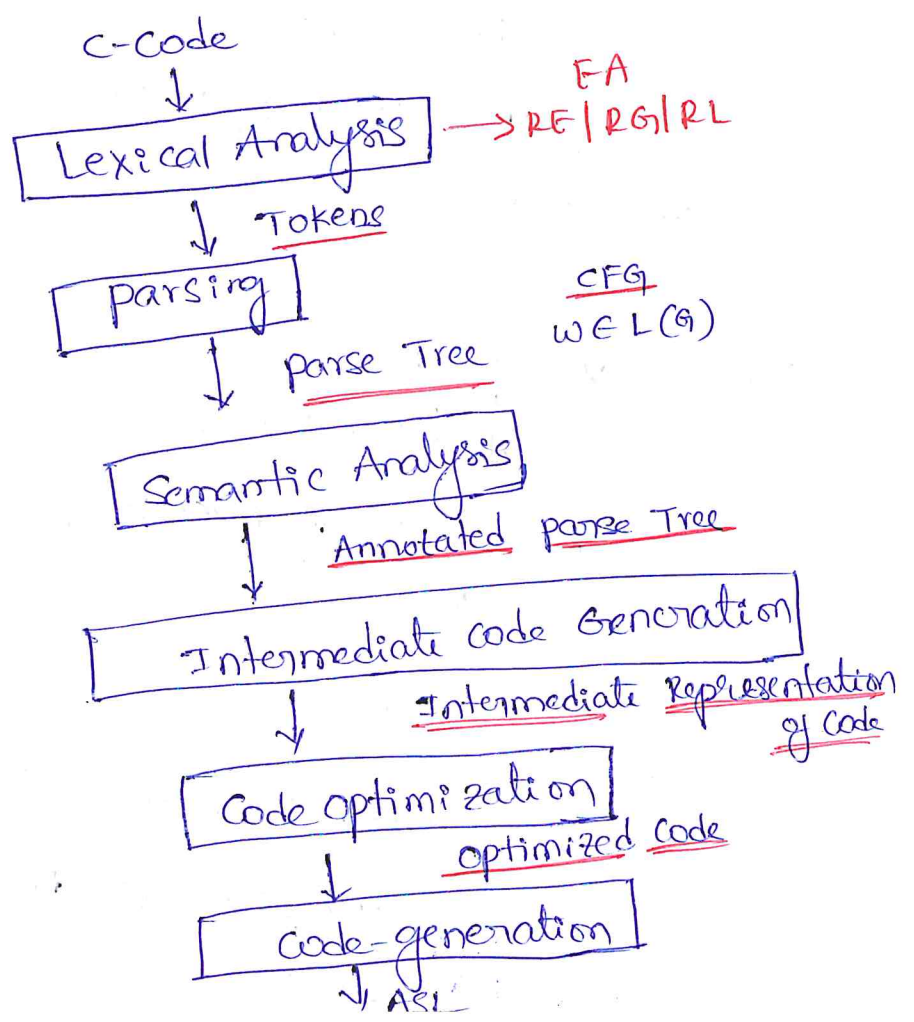


Load all necessary files into memory.

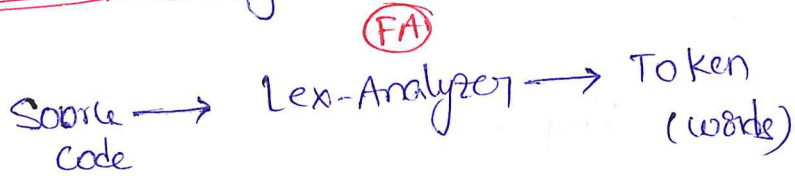
Translates : Assemblers, Compilers



Phases of a Compiler:



Lexical-Analysis:



peeling an Onion

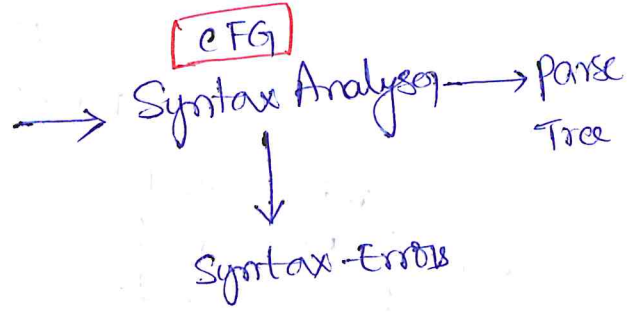


int x=10; → EOS
 KW id op constant

KW: Keyword
 Symbol / char
id: identifier
 op: operator

Parsing / Syntax Analysis:

kw id op constant EOS
 ───────────
 Tokens



c-prog: CFG

int 10=x;

kw const op id EOS

w ∈ L(G)? → decidable
 ⇓
 CYK

java: CFG

Eg:

x = a + b;

↓ Lex-Analyzer

id eq op id op id; (Tokens)
 w

CFG:

S → id eq op A

A → id op id;

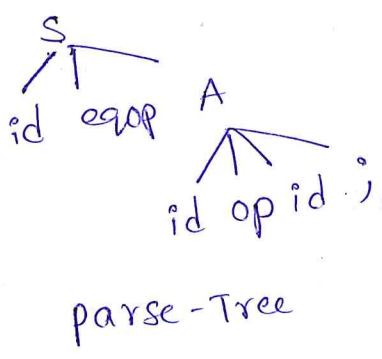
Σ = {id, eq, op, ;}

w ∈ L(G)?

S → id eq op A

→ id eq op id op id;

→ w



Semantic Analysis:

- Type-checking
- Type Conversion
- variable-declared?

```

int x=10;
bool b=TRUE;
char c='a';
int x=b+c;
      ↓   ↓
      x  ASCII('a')
  
```

bool → int x

Intermediate Code-Gen:

Generate IC to make the next-phase easier

x = a + b * c ;

IC t₁ = b * c ;

t₂ = a + t₁ ;

x = t₂ ;

Code-optimization:

- < M/c indep optimization
- < M/c dep optimization

Reduce the # of instructions in the code without impacting the output.

```

int x = y || 1;
    ↓
int x = y | 1;
    
```

} Short-Circuiting

Microprocessor

Front-End: Lexical Analysis → Machine Independent optimization

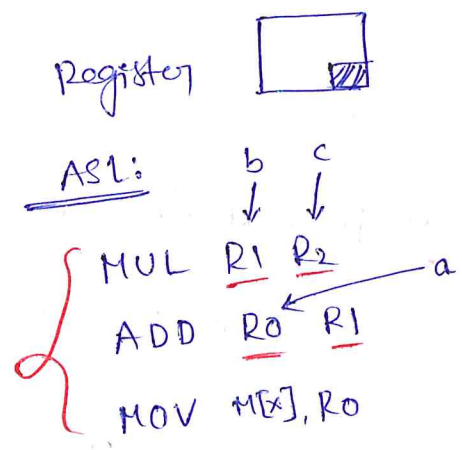
Back-End: Machine dep optimization to code-gen



Code Generation:

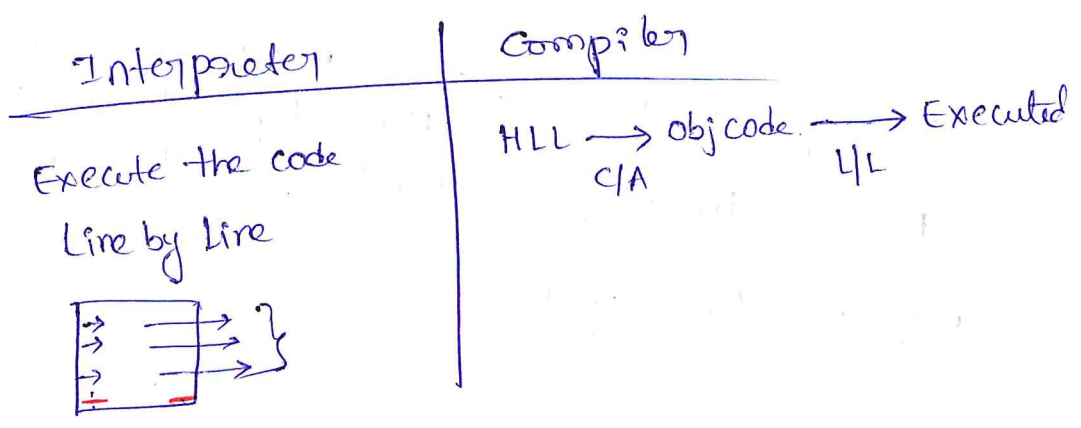
```

x = a + b * c;
    ↓
t1 = b * c;
t2 = a + t1;
x = t2
    
```



R1: b * c
R0: a + b * c

code-optimization is optional



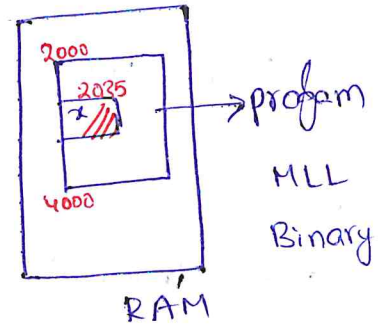
8)

| Symbol Name | Type | Scope |
|-------------|------------------|--------------------|
| bar | junction, double | extern |
| x | double | junction parameter |
| foo | junction, double | global |
| Coont | int | junction parameter |
| sum | double | block local |
| i | int | for-loop statement |

C-prog

Symbol Table

1. Name
2. Type
3. Scope
4. Size
5. offset
6. Misc



program
 start: 2000
 End: 4000
 x: 2035, 2036

Implementation of Symbol Table:

- ① Linear-Table
- ② List
- ③ Tree
- ④ Hash Table

$$\text{offset}(x) = 2035 - 2000 = 35$$

operations:

- ① Insert / ~~lookup~~
- ② search / lookup
- ③ Modify
- ④ Delete

```

{ int i;
  }
{ float i;
  }

```

Error-Handler:

Warnings, Errors

① Lexical Errors: `nti x=10;` `int x=10;`
↓
Not kw

② Syntax - Errors: `int x=10` ↓ `int x=10;`
No semi-colon
`if (x > y)` `(x > y) if`
{ } { }
↓ ↓
{ }

③ Semantic-Errors:
`int x = "abc";` → Array of char
↓ ↓ ↓ ↓
kw id eqop const;

`int x=10;`
`int y=20;`
`z=x+y;`
Not declared

④ Fatal Errors: → Memory insufficiency → Runtime

Lexical Analysis: A deep dive

int x;
Grouping of characters into words

| <u>Lexeme</u> | <u>Token</u> |
|---------------|--------------|
| int | keyword |
| x | id |
| ; | EOS |

int x;
kw id op

↓ parser / Syntax Analyzer

double
PI = 3.1415;

| <u>lexeme</u> | <u>Token</u> |
|---------------|--------------|
| double | kw |
| PI | id |
| = | op |
| 3.1415 | Const |
| ; | op |

Secondary / Additional Task of Lexical Analyzer:

- ① Remove comments // -----
/* -----
----- */
- ② Remove white space char
blanks, Tabs
- ③ Correlate the error with the line number

1: int x



LA



1: kw id



Error

Error in Line number 1

Building a Lexical Analyser:



Simple-string

Pattern-matching System - Regular Expression



Regular Languages



email-address

—@—. —

Moore/Mealy Machine

① Write your own code

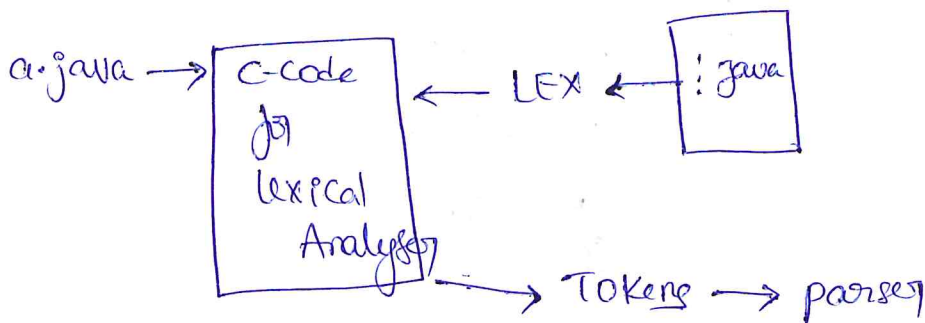
Tables/Arrays: kw, op ...

Rules: Identifiers, Constant

② LEX (Unix, Linux, Mac)

↳ Special purpose tool

→ Give the Rules in a specific format



YACC

↓

Syntax Analysis

Lex-tool Tutorial: <https://youtu.be/5UbolqaHAfk>Syntax - Analysis / Parsing:

↳ Most of this course

CFG, parse-tree, derivation

↑

TOC, CYK, $W \in L(G)?$

Grammar is a 4 Tuple

$$G = \langle V, T, P, S \rangle$$

V: set of Variable / Non-terminals

T: Set of TerminalsP: production rules setS: start-symbol $\in V$

$$P = \{ E \rightarrow E + E$$

$$E \rightarrow E * E$$

$$E \rightarrow E = E$$

$$E \rightarrow id$$

}

$$V = \{ E \}$$

$$T = \{ id, +, *, = \}$$

$$S = \{ E \}$$

Lex-Analyzer LEX

Semantic Analysis

CFG

CO

CG

Derivation:

$$G: E \rightarrow E = E \mid E + E \mid E * E \mid id$$

$$w(\text{String}) = id_1 + id_2 * id_3$$

$$w \in L(G)?$$

(Start-symbol)

Derivation

$$\begin{aligned}
 & E \rightarrow E + E \\
 & \rightarrow E + E * E \\
 & \rightarrow id + id * id = w
 \end{aligned}$$

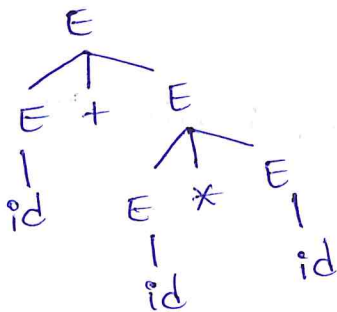
$$2 + 3 * 6;$$

↓ Lex Analysis

$$id_1 + id_2 * id_3$$

↓ Syntax Analysis

Derivation Tree/parse Tree:



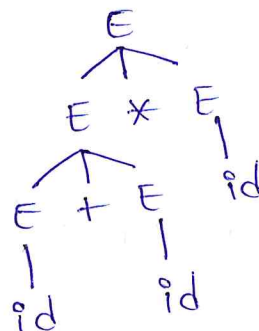
Pictorial Representation of parse tree (or) derivation

Types of derivation → Left most derivation
→ Right most derivation

$$G: E \rightarrow E * E \mid E + E \mid E = E \mid id$$

$$w = id + id * id \quad w \in L(G)$$

LMD

$$\begin{aligned}
 E & \rightarrow \underline{E} * E \\
 & \rightarrow \underline{E + E} * E \\
 & \rightarrow id + id * E \\
 & \rightarrow id + id * id
 \end{aligned}$$


LMDT

$$E \rightarrow E + E$$

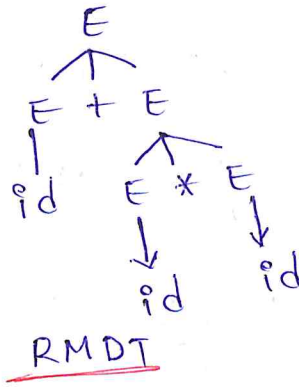
$$\rightarrow E + E * E$$

$$\rightarrow E + E * id$$

$$\rightarrow E + id * id$$

$$\rightarrow id + id * id = w$$

RMD



RMDT

Classification of CFG:

Not REC-Lang (TM)
↳ TTM



Recursive Grammar:

① $S \rightarrow Sa|b$

∃ at least one production having same variable on LHS & RHS

② $S \rightarrow as|b$

③ $S \rightarrow asb|e$

$L(RCFG) = \text{Infinite}$

Non-Recursive Grammar:

① $S \rightarrow AaB$

$A \rightarrow b|c$

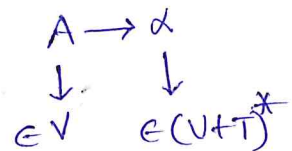
$B \rightarrow c|d$

②

$S \rightarrow AB|BA$

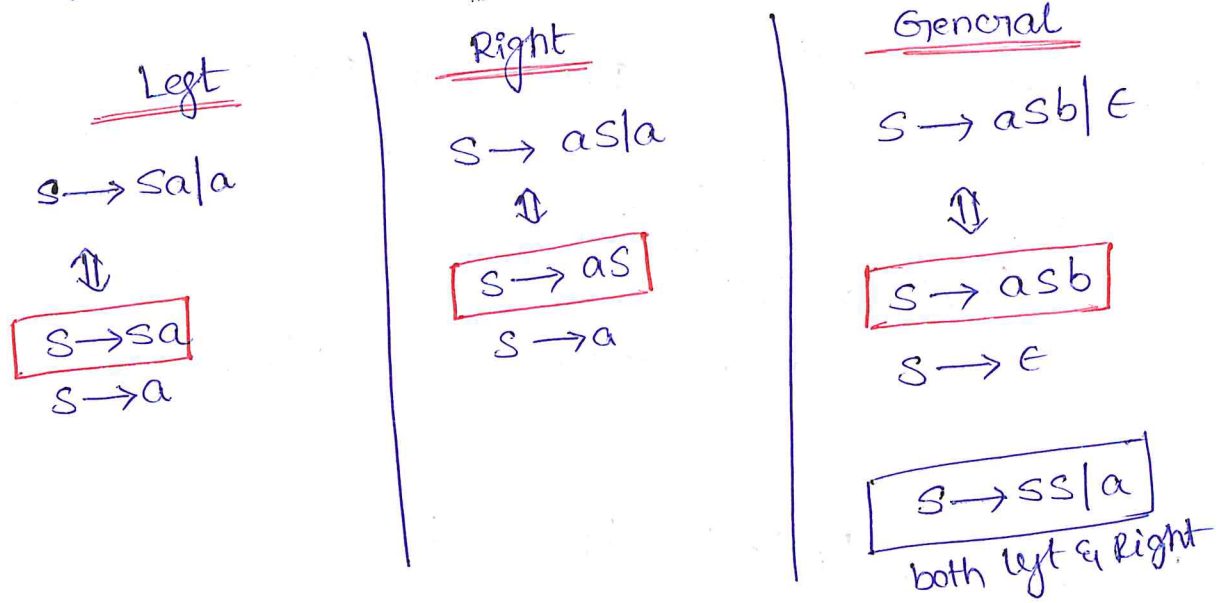
$A \rightarrow aB|b$

$B \rightarrow b|c$

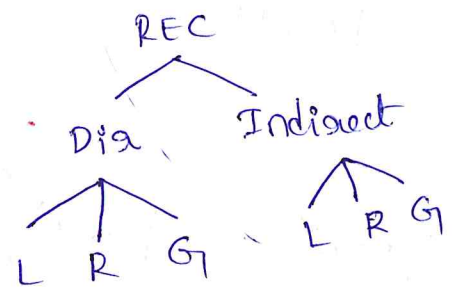
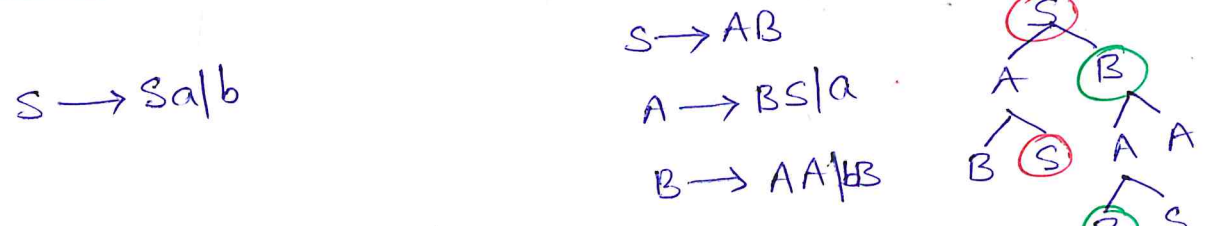


$L(NRCFG) = \text{Finite}$

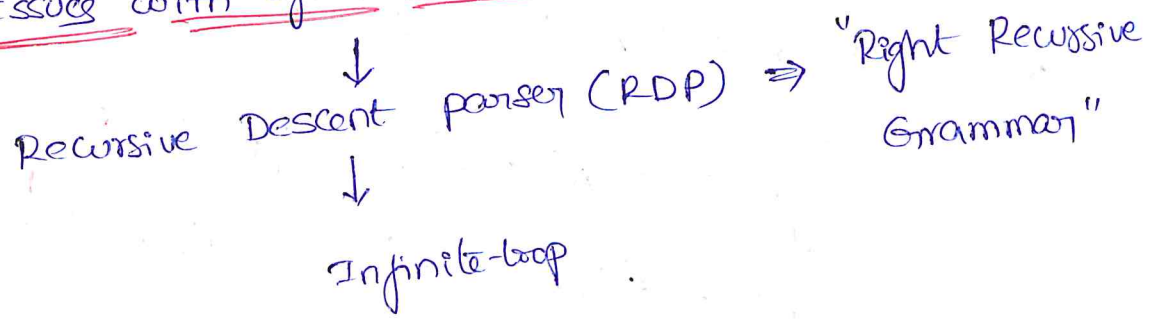
Types of Recursion:



Direct-Recursion vs Indirect-Recursion:



Issues with left-Recursive Grammar:



Conversion from left-Recursion to Right Recursion

① $A \rightarrow A\alpha \mid B$ $B, B\alpha, B\alpha\alpha, B\alpha\alpha\alpha, \dots$
 $A \rightarrow B\alpha^*$

$$\Rightarrow \left\{ \begin{array}{l} A \rightarrow BA' \\ A' \rightarrow \epsilon \mid \alpha A' \end{array} \right\}$$

Right Recursive Grammar

② $A \rightarrow A\alpha \mid B_1 \mid B_2 \mid \dots \mid B_n$ LRG

$$\Rightarrow \left\{ \begin{array}{l} A \rightarrow B_1 A' \mid B_2 A' \mid \dots \mid B_n A' \\ A' \rightarrow \epsilon \mid \alpha A' \end{array} \right\} \text{ RRG}$$

③ $A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n \mid B$

$$\left\{ \begin{array}{l} A \rightarrow BA' \\ A' \rightarrow \epsilon \mid \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_n A' \end{array} \right\} \text{ RRG}$$

④ $A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_n \mid B_1 \mid B_2 \mid \dots \mid B_n$

$$\left\{ \begin{array}{l} A \rightarrow B_1 A' \mid B_2 A' \mid \dots \mid B_n A' \\ A' \rightarrow \epsilon \mid \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_n A' \end{array} \right\} \text{ RRG}$$

Eg ①:

$$A \rightarrow Aab \mid \frac{c}{\alpha} \mid \frac{\epsilon}{\beta}$$

$$\boxed{\begin{array}{l} A \rightarrow cA' \\ A' \rightarrow abA' \mid \epsilon \end{array}} \text{ PRG}$$

②

$$A \rightarrow AaB \mid b$$

$$B \rightarrow Ba \mid c$$

$$\Rightarrow \boxed{\begin{array}{l} A \rightarrow bA' \\ A' \rightarrow \epsilon \mid aBA' \\ B \rightarrow cB' \\ B' \rightarrow \epsilon \mid aB' \end{array}} \text{ PRG}$$

③

$$A \rightarrow A(A) \mid \frac{0}{\alpha} \mid \frac{\epsilon}{\beta}$$

$$\Sigma = \{(,), 0\}$$

$$\Rightarrow \boxed{\begin{array}{l} A \rightarrow 0A' \\ A' \rightarrow (A)A' \mid \epsilon \end{array}}$$

④

$$A \rightarrow AA \mid 0$$

$$\Rightarrow \boxed{\begin{array}{l} A \rightarrow 0A' \\ A' \rightarrow AA' \mid \epsilon \end{array}}$$

⑤

$$A \rightarrow A(A)A \mid \frac{0}{\alpha} \mid \frac{\epsilon}{\beta_1 \beta_2}$$

$$\left. \begin{array}{l} A \rightarrow 0A' \mid 1A' \\ A' \rightarrow (A)AA' \mid \epsilon \end{array} \right\} \text{ PRG}$$

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Eg 6:

$$S \rightarrow \frac{SSS|a}{\alpha \quad \beta}$$

$$\left. \begin{array}{l} S \rightarrow aS' \\ S' \rightarrow SSS'| \epsilon \end{array} \right\} \text{RRG}$$

Eg 7:

$$S \rightarrow \frac{SOS| \epsilon}{\alpha \quad \beta}$$

$$\left. \begin{array}{l} S \rightarrow S' \\ S' \rightarrow OS|S'| \epsilon \end{array} \right\} \text{RRG}$$

Eg 8:

$$S \rightarrow \frac{Sasb| \quad Sbsa| \epsilon}{\alpha_1 \quad \alpha_2 \quad \beta}$$

$$\left. \begin{array}{l} S \rightarrow S' \\ S' \rightarrow asbs'|bsas'| \epsilon \end{array} \right\} \text{RRG}$$

Eg 9:

$$S \rightarrow SOS| \epsilon$$

$$\left. \begin{array}{l} S \rightarrow IS' \\ S' \rightarrow OSS'| \epsilon \end{array} \right\} \text{RRG}$$

Eg 10:

$$S \rightarrow \frac{Sa| \quad aS|b}{\alpha \quad \beta_1 \quad \beta_2}$$

$$\Rightarrow \left. \begin{array}{l} S \rightarrow Sa|b \\ S \rightarrow aS \end{array} \right\} \Rightarrow$$

$$\left. \begin{array}{l} S \rightarrow bS' \\ S' \rightarrow aS'| \epsilon \\ S \rightarrow aS \end{array} \right\} \text{RRG}$$

Eg 11:

$$E \rightarrow \frac{E+E| \quad E \times E| \quad id}{\alpha_1 \quad \alpha_2 \quad \beta}$$

$$\Rightarrow \left. \begin{array}{l} E \rightarrow id E' \\ E' \rightarrow +EE'| \times EE'| \epsilon \end{array} \right\}$$

Eg 12:

$$E \rightarrow E+T \mid T$$

$$T \rightarrow id$$

$$\Rightarrow \left. \begin{aligned} E &\rightarrow TE' \\ E' &\rightarrow +TT' \mid \epsilon \\ T &\rightarrow id \end{aligned} \right\} \text{RRG}$$

Eg 13:

$$E \rightarrow E+T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

$$\left. \begin{aligned} E &\rightarrow TE' \\ E' &\rightarrow +TE' \mid \epsilon \\ T &\rightarrow FT' \\ T' &\rightarrow *FT' \mid \epsilon \\ F &\rightarrow (E) \mid id \end{aligned} \right\} \text{RRG}$$

Eg 14:

$$S \rightarrow (L) \mid a$$

$$L \rightarrow L, S \mid S$$

$$\left. \begin{aligned} S &\rightarrow (L) \mid a \\ L &\rightarrow SL' \\ L' &\rightarrow ,SL' \mid \epsilon \end{aligned} \right\} \text{RRG}$$

Eg 15:

$$R \rightarrow \underline{RR} \mid \underline{Ra} \mid aR \mid \frac{b}{R}$$

$$R \rightarrow aR$$

$$R \rightarrow bR'$$

$$R' \rightarrow \epsilon \mid RR' \mid aR'$$

RULE 3
 RRG

Eg 16:

$$S \rightarrow AaB$$

$$A \rightarrow aA \mid Ba$$

$$B \rightarrow AB \mid b$$

Indirect
Recursion

$$S \rightarrow AaB$$

$$A \rightarrow aA \mid ba \mid \underline{A}Ba$$

$$B \rightarrow AB \mid b$$

(8)

$$S \rightarrow AaB$$

$$A \rightarrow aA \mid Ba$$

$$B \rightarrow aAB \mid \underline{Ba}B \mid b$$

$$S \rightarrow AaB$$

$$A \rightarrow aA \mid \underline{ba} \mid \underline{A}Ba$$

$$B \rightarrow AB \mid b$$

⇓

$$S \rightarrow AaB$$

$$A \rightarrow aA$$

$$A \rightarrow baA'$$

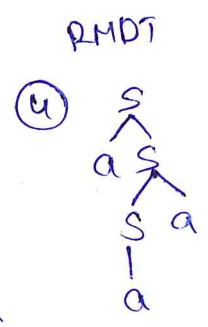
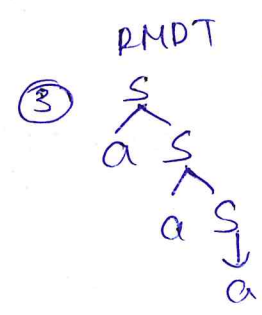
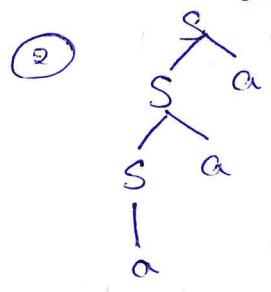
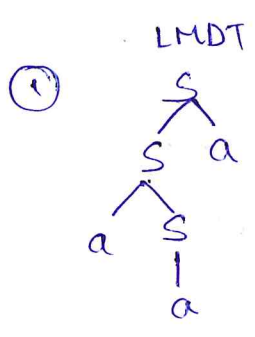
$$A' \rightarrow \epsilon \mid BaA'$$

$$B \rightarrow AB \mid b$$

Ambiguous and Unambiguous Grammar:

$$G: S \rightarrow sa|as|a^3$$

w = a^3



Ambiguous:
 \exists more than one derivation for any word $w \in L(G)$

unambiguous Grammar:

\exists only one derivation for all words $w \in L(G)$

properties:

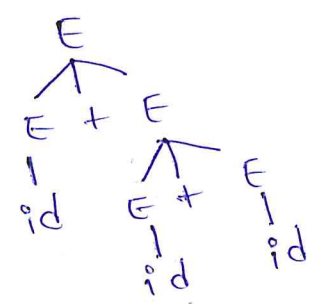
- ① Unambiguous Grammar \Rightarrow (LMDT = RMDT)
- ② Grammar: Left rec & Right rec \Rightarrow Ambiguous
- ③ Ambiguity of CFG is undecidable.
 $\Rightarrow \nexists$ an algorithm to determine ambiguity of CFG

Eliminating Ambiguity:

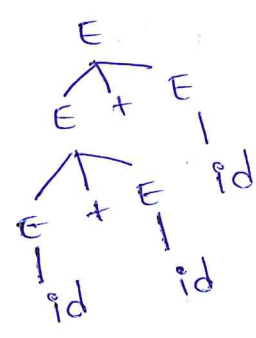
$G_1 \left\{ \begin{array}{l} E \rightarrow E + E \mid id \\ w = id + id + id \end{array} \right.$

$\Rightarrow \left. \begin{array}{l} E \rightarrow E + T \mid T \\ T \rightarrow id \end{array} \right\} G_2$ unambiguous

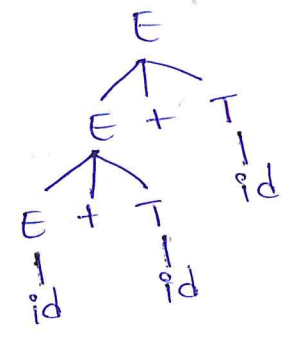
$L(G_1) = L(G_2)$



\neq



$w = id + id + id$



Ambiguous & not:

① $S \rightarrow \underline{S} \underline{O} \underline{S} | \epsilon$ - Ambiguous

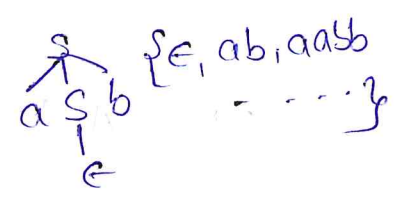
② $S \rightarrow \underline{S} \underline{a} \underline{S} | \underline{S} \underline{b} \underline{S} | \epsilon$ - Ambiguous

③ $S \rightarrow \underline{S} \underline{S} \underline{S} | 0$ - Ambiguous

④ $S \rightarrow \underline{S} \underline{O} \underline{S} \underline{I} \underline{S} | 1$ - Ambiguous

⑤ $S \rightarrow E + E | E * E | id$ - Ambiguous

⑥ $S \rightarrow a S b | \epsilon$ - unambiguous



⑦ $S \rightarrow SAB$
 $A \rightarrow aAB | b$
 $B \rightarrow BS | a$

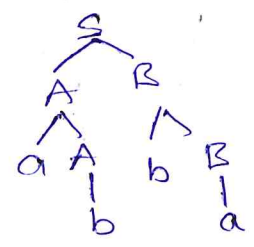
⑧ $S \rightarrow \underline{S} \underline{A} \underline{B} \underline{S} | SAa$ - Ambiguous

$A \rightarrow aAB | b$
 $B \rightarrow BS | a$

⑨ $A \rightarrow (A) | AA | 0$ - Ambiguous

⑩ $S \rightarrow AB$ - unambiguous

$A \rightarrow aA | b$
 $B \rightarrow bB | a$



⑪ Regular Grammar : unambiguous

CFL / CFG : Ambiguous or Unambiguous

Left-Factoring: Transformation

→ LR(1) to PRG

→ Ambiguous to unambiguous

$$A \rightarrow \alpha\beta_1 | \alpha\beta_2 | \alpha\beta_3$$

$$\alpha \in (V \cup T)^*$$

Same prefix

$$\beta_i \in (V \cup T)^*$$

$$\Rightarrow \boxed{\begin{array}{l} A \rightarrow \alpha\beta \\ B \rightarrow \beta_1 | \beta_2 | \beta_3 \end{array}}$$

① $A \rightarrow \underline{aAb} | \underline{aAd} | e$

↓ LF

$$\boxed{\begin{array}{l} A \rightarrow aAB | e \\ B \rightarrow b | d \end{array}}$$

② $A \rightarrow (A)A | \underline{(A)Ab} | \underline{(A)Aba} | e | f$

↓

$$A \rightarrow \underline{(A)A} | \underline{(A)Ab}B | e | f$$

$$B \rightarrow e | a$$

↓

$$\boxed{\begin{array}{l} A \rightarrow (A)Ac | e | f \\ B \rightarrow e | a \\ C \rightarrow e | bB \end{array}}$$

3

$A \rightarrow abd | abde | \underline{abdA} | \underline{abdAe} | f | g$

LF \Downarrow

$A \rightarrow \underline{abd} | \underline{abde} | A \cdot \underline{abdAB} | \cdot | f | g$

$B \rightarrow e | e$

\Downarrow

| |
|--------------------------------------|
| $A \rightarrow abdc \cdot f g$ |
| $C \rightarrow e e AB$ |
| $B \rightarrow e e$ |

Non-deterministic & Deterministic Grammar:

TOC $\left\{ \begin{array}{l} DCFG \rightarrow DCFL \rightarrow DPDA \\ NCFG \rightarrow CFL \rightarrow NPDA/PDA \\ \quad \quad \quad (NCFL) \end{array} \right.$

Note: If G is DCFG then G is unambiguous

Dangling-Else Ambiguity:

$stmt \rightarrow if\ expr\ then\ stmt |$

$if\ expr\ then\ stmt\ else\ stmt |$

other

$w \neq if\ E_1\ then\ if\ E_2\ then\ S_1\ else\ S_2$

\Downarrow
2-parse tree

Syntactically correct

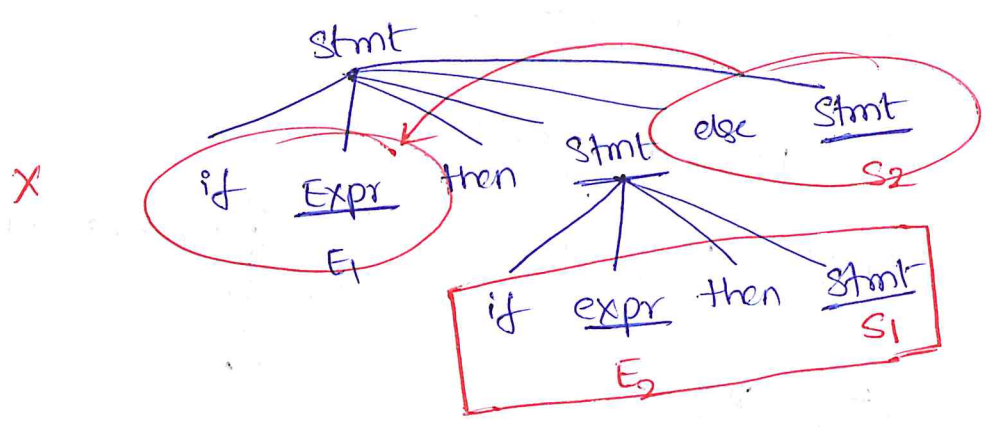
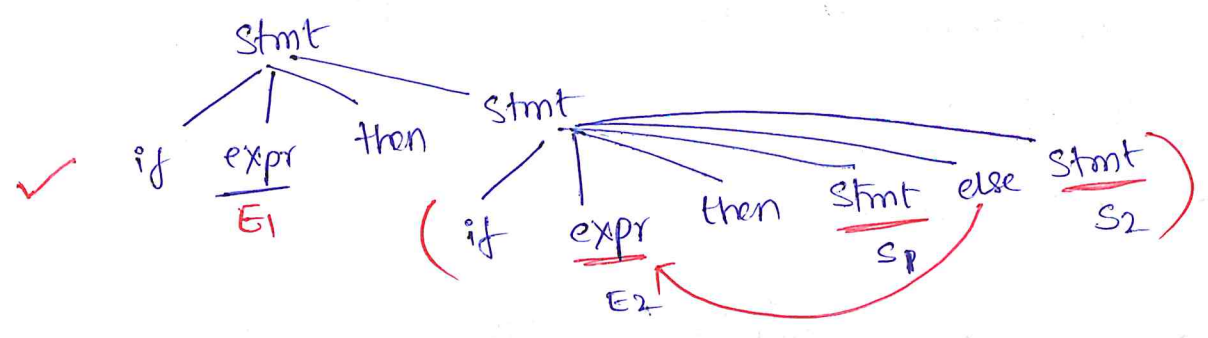
Source Code

\downarrow lex-analysis

tokens

\downarrow parser

Ambiguity:



Most programming languages support else will belong to the nearest/closest unmatched if statement.

Eliminating Ambiguity:

stmt → matched-stmt
| open-stmt

matched-stmt → if expr then matched-stmt
| else matched-stmt
| other

open-stmt → if expr then stmt
| if expr then matched-stmt else open-stmt

$w = \text{if } E_1 \text{ then if } E_2 \text{ then } S_1 \text{ else } S_2$

The grammar will generate unique parse tree for the given string.

G is unambiguous.

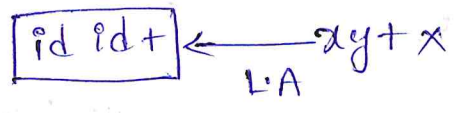
Grammar for programming languages:

① if... else... production-only

② $E \rightarrow E + E \mid E * E \mid E - E \mid E / E \mid \text{id}$

id + id x + y x + y + z

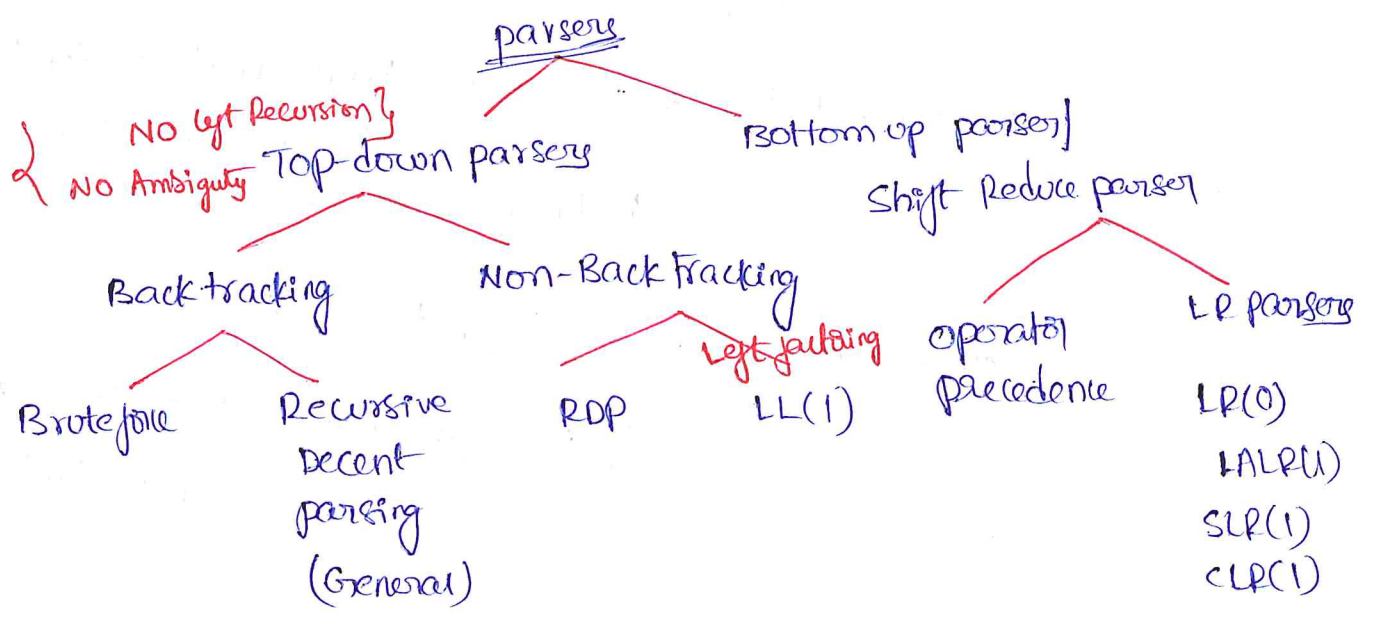
$x + y * z$
↓ lex-Analyser
id + id * id
↓



③ C-programming language

↓
parser CFG

URL: <https://www.lysator.liu.se/c/ANSI-C-grammar-y.html>



Top-down parser:

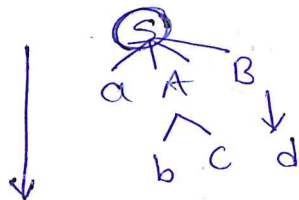
Free from Left-Recursion
Left-Recursive

① $S \rightarrow aAB$

$A \rightarrow bc|b$

$B \rightarrow d$

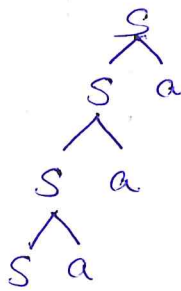
let $w = abcd$



parse-Tree

Left-most derivation
Tree

② $S \rightarrow \underline{S}a|a$

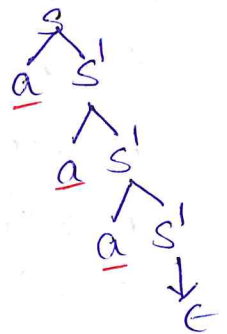


$w = aaa$

Unambiguous

③ $S \rightarrow as'$
 $s' \rightarrow \epsilon|as'$

$w = \underline{aaa}$



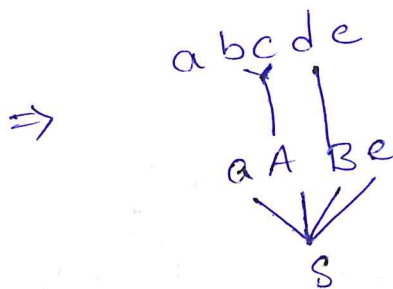
Bottomup parsing:

$S \rightarrow aABc$

$A \rightarrow bc$

$B \rightarrow d$

$w = abcde$



Take the
input string
and derive
the start
symbol of
the grammar.

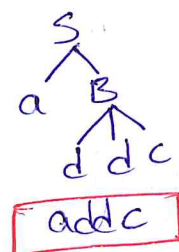
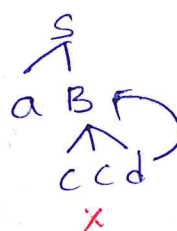
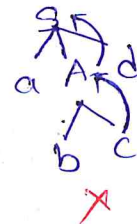
Backward Force: (TDP)

$S \rightarrow aAd|aB$

$A \rightarrow b|bc$

$B \rightarrow ccd|ddc$

$w = \underline{a}d\underline{d}c$



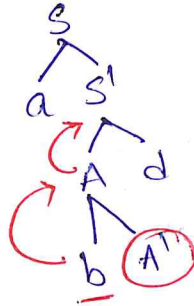
Bottom-Up \Rightarrow Try all possibilities

Recursive Descent parser: (General)

- $S \rightarrow aS'$
- $S' \rightarrow Ad|B$
- $A \rightarrow bA'$
- $A' \rightarrow c|E$
- $B \rightarrow ccd|ddc$

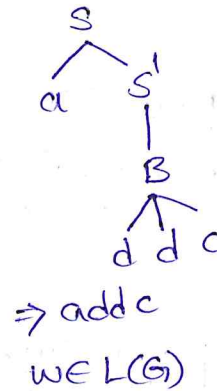
May need
back track

Slightly
Smarter than
Bottom-Up



$w = addc$

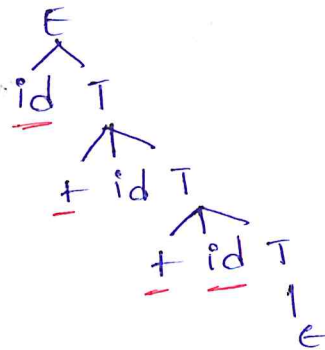
No Left-Recursion, unambiguous



Recursive Descent parser (program)

- $E \rightarrow idT$
- $T \rightarrow +idT|E$
- $w = id+id+id$

(No backtracking
here)



Left-factored Grammar + RDP:

- $S \rightarrow aAd|AB$
- $A \rightarrow b|bc$
- $B \rightarrow ccd|ddc$

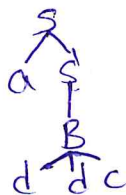
$w = addc$

- $S \rightarrow aS'$
- $S' \rightarrow Ad|B$
- $A \rightarrow bA'$
- $A' \rightarrow c|E$
- $B \rightarrow ccd|ddc$

\Downarrow

Unambiguous + No Left Recursion
No Same prefixes

addc



left factoring simplifies in choosing the productions.

LL(1) parser: FIRST(X), Follow(A)

FIRST(X) = Set of all terminals (a), s.t

$X \rightarrow aB \rightarrow$ string of terminals & Variables
 \downarrow
 terminal

① $S \rightarrow a|b|\epsilon$

$First(S) = \{a, b, \epsilon\}$

② $S \rightarrow aA|\epsilon$

$First(S) = \{a, \epsilon\}$

$A \rightarrow b$

$First(A) = \{b\}$

③ $S \rightarrow aA|bB$

$First(S) = \{a, b\}$

$A \rightarrow AB|c$

$First(A) = \{c\}$

$B \rightarrow CB|d$

$First(B) = \{c, d\}$

④ $S \rightarrow AB$

$First(S) = First(A)$

$A \rightarrow a|b$

$= \{a, b\}$

$B \rightarrow d$

$First(B) = \{d\}$

$S \xrightarrow{*} aB$

$S \rightarrow AB$

$\underbrace{aB} \quad bB$

$S \xrightarrow{*} bB$

⑤ $S \rightarrow AB$

$First(S) = \{a, c, d\}$

$A \rightarrow a|\epsilon$

$First(A) = \{a, \epsilon\}$

$B \rightarrow c|d$

$First(B) = \{c, d\}$

$S \rightarrow AB$

$\xrightarrow{*} aB$

$\xrightarrow{*} B$

$\{c, d\}$

⑥ $S \rightarrow AB$ $\text{First}(S) = \{a, b, \epsilon\}$
 $A \rightarrow aA | \epsilon$ $\text{First}(A) = \{a, \epsilon\}$
 $B \rightarrow bB | \epsilon$ $\text{First}(B) = \{b, \epsilon\}$

⑦ $S \rightarrow AaB$ $\text{First}(S) = \{b, a\}$
 $A \rightarrow bA | \epsilon$ $\text{First}(A) = \{b, \epsilon\}$
 $B \rightarrow cB | d$ $\text{First}(B) = \{c, d\}$

⑧ $S \rightarrow ABb$ $\text{First}(S) = \text{First}(A) = \{b, d\}$
 $A \rightarrow Ba | \epsilon$ $\text{First}(A) = \{\epsilon, b, d\}$
 $B \rightarrow b | d$ $\text{First}(B) = \{b, d\}$

⑨ $S \rightarrow AaB | Ba$ $\text{First}(S) = \{a, b, \epsilon\}$
 $A \rightarrow Ba | b$ $\text{First}(A) = \{a, b\}$
 $B \rightarrow aB | \epsilon$ $\text{First}(B) = \{a, \epsilon\}$

⑩

| | |
|--------------------------------|---------------------------------------|
| $S \rightarrow AaB \epsilon$ | <u>Sym</u> <u>First</u> |
| $A \rightarrow BbA a$ | S $\{\epsilon, f, d, a, \epsilon\}$ |
| $B \rightarrow Db d$ | A $\{\epsilon, f, d, a\}$ |
| $D \rightarrow \epsilon f$ | B $\{\epsilon, d, d\}$ |
| | D $\{\epsilon, f\}$ |

⑪

| | |
|----------------------------------|---------------------------|
| $E \rightarrow TE'$ | <u>Sym</u> <u>First</u> |
| $E' \rightarrow +TE' \epsilon$ | E $\{\epsilon, id\}$ |
| $T \rightarrow FT'$ | E' $\{+, \epsilon\}$ |
| $T' \rightarrow *FT' \epsilon$ | T $\{c, id\}$ |
| $F \rightarrow (E) id$ | T' $\{*, \epsilon\}$ |
| | F $\{c, id\}$ |

12) $S \rightarrow (L) | a$
 $L \rightarrow SL'$
 $L' \rightarrow \epsilon | SL' | \epsilon$

| | First |
|----|----------------|
| S | {(, a} |
| L | {(, a} |
| L' | {, , \epsilon} |

13) $S \rightarrow ABCDE$
 $A \rightarrow a | \epsilon$
 $B \rightarrow b | \epsilon$
 $C \rightarrow c | \epsilon$
 $D \rightarrow d | \epsilon$
 $E \rightarrow \epsilon$

| | First |
|---|------------------------|
| S | {a, b, c, d, \epsilon} |
| A | {a, \epsilon} |
| B | {b, \epsilon} |
| C | {c, \epsilon} |
| D | {d, \epsilon} |
| E | {\epsilon} |

14) $S \rightarrow AaAb | BbBa$
 $A \rightarrow \epsilon$
 $B \rightarrow \epsilon$

| | First |
|---|------------|
| S | {a, b} |
| A | {\epsilon} |
| B | {\epsilon} |

Follow(A): Set of terminals (α) s.t
 $S \xrightarrow{*} \alpha A \alpha' \rightarrow (V+T)^*$
 Variable \downarrow
 $(V+T)^*$ var Terminal

$\alpha \in \text{Follow}(A)$

$w = abab\$$
 \hookrightarrow To represent end of the string

2) $S \rightarrow \alpha A$
 \downarrow

Then $\text{Follow}(A) = \text{Follow}(S)$

3) If S is the Start Symbol of the Grammar

then add '\$' to the follow(s).

① $S \rightarrow a|b|e \Rightarrow \text{Follow}(S) = \{\$ \}$ let $w = a\$$

② $S \rightarrow aA\underline{b}$ $\Rightarrow \text{Follow}(S) = \{\$ \}$
 $A \rightarrow \underline{A}a|c$ $\text{Follow}(A) = \{a, b\}$
left-recursive Grammar

③ $S \rightarrow asb|e \Rightarrow \text{Follow}(S) = \{\$, b\}$

④ $S \rightarrow aS|sa|e \Rightarrow \text{Follow}(S) = \{\$, a\}$

⑤ $S \rightarrow aA \Rightarrow \text{Follow}(S) = \{\$ \}$
 $A \rightarrow b \text{ Follow}(A) = \{\$ \}$



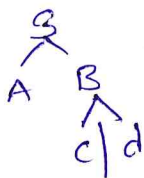
⑥ $S \rightarrow aA$ $\Rightarrow \text{Follow}(S) = \{\$ \}$
 $A \rightarrow aAb|c \text{ Follow}(A) = \{\$, b\}$

⑦ $A \rightarrow (A)|a \Rightarrow \text{Follow}(A) = \{\$,)\}$

⑧ $S \rightarrow \underline{AB}$ $\Rightarrow \text{Follow}(S) = \{\$ \}$
 $A \rightarrow a|b \text{ Follow}(A) = \{c, d\}$
 $B \rightarrow c|d \text{ Follow}(B) = \{\$ \}$

$S \rightarrow \underline{AB}$

$\text{First}(B) = \{c, d\}$



9

$S \rightarrow AB$
 $A \rightarrow a|b|\epsilon$
 $B \rightarrow c|d$

| | Follow |
|---|----------|
| S | { \$ } |
| A | { c, d } |
| B | { \$ } |

10

$S \rightarrow AB$
 $A \rightarrow a|b$
 $B \rightarrow c|d|\epsilon$
 $S \rightarrow A\epsilon \rightarrow A$

| | Follow |
|---|--------------|
| S | { \$ } |
| A | { c, d, \$ } |
| B | { \$ } |

11

$S \rightarrow SOSIS2|\epsilon \Rightarrow \text{Follow}(S) = \{ \$, 0, 1, 2 \}$

12

$S \rightarrow ABC$
 $A \rightarrow a|\epsilon$
 $B \rightarrow b|\epsilon$
 $C \rightarrow d|\epsilon$

| | Follow | ABC |
|---|--------------|-----|
| S | { \$ } | |
| A | { \$, b, d } | |
| B | { \$, d } | |
| C | { \$ } | |

13

$S \rightarrow AA$
 $A \rightarrow aA|b$

| | Follow |
|---|--------------|
| S | { \$ } |
| A | { \$, a, b } |

$S \rightarrow AA$
 $\text{First}(A) = \{ a, b \}$

14

$E \rightarrow TE'$
 $E' \rightarrow +TE'|\epsilon$
 $T \rightarrow FT'$
 $T' \rightarrow *FT'|\epsilon$
 $F \rightarrow (E)|id$

| | Follow |
|----|-----------------|
| E | { \$,) } |
| E' | { \$,) } |
| T | { +, \$,) } |
| T' | { +, \$,) } |
| F | { *, +, \$,) } |

$E' \rightarrow +TE'|\epsilon$
 \downarrow
 $E' \rightarrow +TE$
 $E' \rightarrow +T$
 $T' \rightarrow *FT'|\epsilon$
 $\Rightarrow T' \rightarrow *FE$
 $T' \rightarrow *F$

15

$S \rightarrow (L) | a$

$L \rightarrow SL'$

$L' \rightarrow , SL' | \epsilon$

| | Follow |
|----|--------------|
| S | { \$, ,, } } |
| L | {) } |
| L' | {) } |

$L \rightarrow SL'$

$First(L') = { ,, \epsilon }$

$L \rightarrow SE$

$L \rightarrow S$

16

$S \rightarrow ABCDE$

$A \rightarrow a | \epsilon$

$B \rightarrow b | \epsilon$

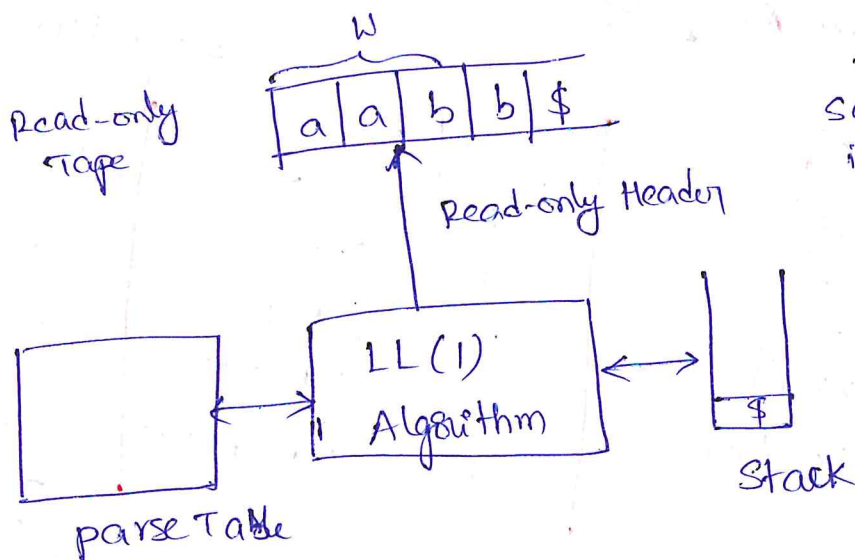
$C \rightarrow c | \epsilon$

$D \rightarrow d | \epsilon$

$E \rightarrow \epsilon$

| | Follow |
|---|-----------------|
| S | { \$ } |
| A | { \$, b, c, d } |
| B | { \$, c, d } |
| C | { \$, d } |
| D | { \$ } |
| E | { \$ } |

LL(1) parser!



- ① Free From Ambiguity
- ② Free From Left-Recursion
- ③ Free From Common-prefixes.

LRDP: Recursion
 ↓
 internal stack

parse-Table

- ① $A \rightarrow (A) | a$
- $A \rightarrow \underline{(A)} \quad \textcircled{1}$
- $A \rightarrow \underline{a} \quad \textcircled{2}$

| | Terminals | | | | |
|---|-----------|---|---|---|----|
| | T | (|) | a | \$ |
| A | | ① | | ② | |

Empty cells represent error

- ① place $A \rightarrow \alpha$ in $T[A, a] \quad \forall a \in \text{First}(\alpha)$
- ② If $\text{First}(\alpha)$ contains ϵ , then add $A \rightarrow \alpha$ $\forall b \in \text{Follow}(A)$

- ② $S \rightarrow AA \quad \textcircled{1}$
- $A \rightarrow aA | b \quad \textcircled{2} \textcircled{3}$
- $\text{First}(S) = \text{First}(A)$
 $= \{a, b\}$

| | a | b | \$ |
|---|---|---|----|
| S | ① | ① | |
| A | ② | ③ | |

- ③ 1. $E \rightarrow TE'$
- 2. $E' \rightarrow +TE'$
- 3. $E' \rightarrow \epsilon$
- 4. $T \rightarrow FT'$
- 5. $T' \rightarrow *FT'$
- 6. $T' \rightarrow \epsilon$
- 7. $F \rightarrow (E)$
- 8. $F \rightarrow id$

| | | | | | | |
|----|----|---|---|---|---|----|
| | id | + | * | (|) | \$ |
| E | ① | | | ① | | |
| E' | | ② | | | ③ | ③ |
| T | ④ | | | ④ | | |
| T' | | ⑥ | ⑤ | | ⑥ | ⑥ |
| F | ⑧ | | | ⑦ | | |

First(E) = { (, id }

First(E') = { +, ε }

First(T) = { (, id }

First(T') = { *, ε }

First(F) = { (, id }

Follow(E) = { \$,) }

Follow(E') = { \$,) }

Follow(T) = { +, \$,) }

Follow(T') = { +, \$,) }

Follow(F) = { *, +, \$,) }

Parsing Algorithm:

① ②

G: A → (A) | a

| | | | | |
|---|---|---|---|----|
| T | (|) | a | \$ |
| A | ① | | ② | |

let w = ((ca)))

| Stack | input | Action |
|--------------------|-----------------------|-----------|
| X | a | |
| \$ | ((ca)))\$ | push(\$) |
| \$ A | ((ca)))\$ | push(A) ① |
| \$) A | ((ca)))\$ | pop |
| \$) A | (ca)))\$ | push(A) ① |
| \$))A | ((a)))\$ | pop |
| \$))A | (a)))\$ | push(A) ① |
| \$))A | ((a)))\$ | POP |
| \$))A | a)))\$ | push a ② |

| Stack | input | Action |
|-------------------------------------------|-------------------------------------------|--------|
| \$)) a | a)) \$ | pop |
| \$)) |)) \$ | pop |
| \$) |) \$ | pop |
| \$) |) \$ | Accept |

Lets Assume x : Stack top a : Current input Symbol

- ① $x = a \neq \$$ pop & increment the input pointer
- ② $x = a = \$$ Accept
- ③ $T(x, a)$ $x \rightarrow a$, pop x from the Stack and push ' a ' into the Stack in reverse order.

$$w =)a(\Rightarrow w \notin L(G)$$

- ④ $T(x, a) = \text{Empty}$ Syntax Error

Example 2:

- ① $S \rightarrow AA$
- ② $A \rightarrow aA$
- ③ $A \rightarrow b$

| | a | b | \$ |
|--------------|---|---|----|
| S | 1 | 1 | |
| A | 2 | 3 | |
| B | | | |

Let $w = abab$

| Stack | input | Action |
|------------------|-------------------|--------------------------------------|
| \$ | abab\$ | push (c) |
| \$\$ | abab\$ | push AA |
| \$AA | abab\$ | push AA |
| \$AAA | abab\$ | pop & increment |
| \$AA | bab\$ | push b |
| \$Ab | bab\$ | pop & increment |
| \$A | ab\$ | push AA |
| \$AA | ab\$ | push a pop & increment |
| \$A | b\$ | push b |
| \$b | b\$ | pop & increment |
| \$ | \$ | Accept |

Example 3:

- $E \rightarrow TE'$
- $E' \rightarrow +TE'$
- $E' \rightarrow \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT'$
- $T' \rightarrow \epsilon$
- $F \rightarrow (E)$
- $F \rightarrow id$

| Stack | Input | Action |
|--------------------|---------------------------|---------------------------|
| \$E | id + id * id\$ | $E \rightarrow TE'$ |
| \$E'T | id + id * id\$ | $T \rightarrow FT'$ |
| \$E'T'F | id + id * id\$ | $F \rightarrow id$ |
| \$E'T'* | id + id * id\$ | pop |
| \$E'T' | + id * id\$ | $T' \rightarrow \epsilon$ |
| \$E' | + id * id\$ | $E' \rightarrow +TE'$ |
| \$E'T | * id * id\$ | pop |

| Stack | input | Action |
|--------------|------------|------------|
| \$ E' T | id * id \$ | T → FT' |
| \$ E' T' F | id * id \$ | F → id |
| \$ E' T' id | id * id \$ | pop |
| \$ E' T' | * id \$ | T' → * FT' |
| \$ E' T' F * | * id \$ | F → id |
| \$ E' T' id | id \$ | T' → E |
| \$ E' | \$ | E' → ε |
| \$ | \$ | Accept |

LL(1) Grammar : Unambiguous + No Left Recursion + left-factored

① G is a grammar free from ε-productions, then G is LL(1) if $\forall A \rightarrow aB \mid aC \dots$
 $First(\alpha_1) \cap First(\alpha_2) \dots \cap First(\alpha_n) = \emptyset$

② G has ε-productions $\forall A \rightarrow \alpha \mid \epsilon$
 $First(\alpha) \cap Follow(A) = \emptyset$
 then G is LL(1)

③ If G has only one alternative on RHS \forall variable then G is LL(1)

$$\begin{aligned}
 A &\rightarrow \alpha \\
 B &\rightarrow \gamma \\
 C &\rightarrow \beta \\
 &\vdots
 \end{aligned}$$

Eg

① $S \rightarrow a|b|c$ LL(1)

② $S \rightarrow \overset{d_1}{a}A | \overset{d_2}{b}B$ ✓
 $A \rightarrow \overset{d_1}{a}A | \overset{d_2}{b}$ ✓
 $B \rightarrow \overset{d_1}{b}B | \overset{d_2}{c}$ ✓
 LL(1)

③ $S \rightarrow \overset{d_1}{a}Ab | \overset{d_2}{B}a$
 $A \rightarrow aB | bB$
 $B \rightarrow aB | b$
 $First(\alpha_1) = \{a\}$ $\{a\} \cap \{a, b\} \neq \emptyset$
 $First(B) = \{a, b\}$
Not LL(1)

④ $S \rightarrow aAb$
 $A \rightarrow aA | \epsilon$
LL(1)
 $First(aA) = \{a\}$ $Follow(A) = \{b\}$
 $\{a\} \cap \{b\} = \emptyset$

⑤ $S \rightarrow aAb | \epsilon$
 $A \rightarrow aAa | \epsilon$
 $First(S) = \{a, \epsilon\}$ $Follow(S) = \{\$ \}$
 $First(A) = \{a\}$ $Follow(A) = \{a, b\}$
 $\{a\} \cap \{a, b\} \neq \emptyset$
NOT LL(1)

⑥ $S \rightarrow aAb | bB$ ✓
 $A \rightarrow aAb | \epsilon$ ✓
 $B \rightarrow bBb | \epsilon$
NOT LL(1)

⑦ $S \rightarrow AS | dBA$
 $A \rightarrow aAb | bB | \epsilon$
 $B \rightarrow bBa | aA | \epsilon$
 $First(B) = \{a, b\}$ $Follow(B) = \{a\}$
NOT LL(1)

8

$$S \rightarrow aA | bB | db$$

$$A \rightarrow bA | Ba | \epsilon$$

$$B \rightarrow aB | dA | \epsilon$$

$$\text{First}(A) = \{a, b, d\}$$

$$\text{Follow}(A) = \{\$, a\}$$

NOT LL(1)

9

$$S \rightarrow AB$$

$$A \rightarrow aB \quad \underline{\underline{LL(1)}}$$

$$B \rightarrow b$$

Bottom-up parsers: \rightarrow widely used

\hookrightarrow Ambiguous & Unambiguous Grammars

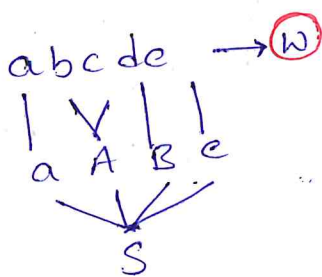
\hookrightarrow Faster

Eg:

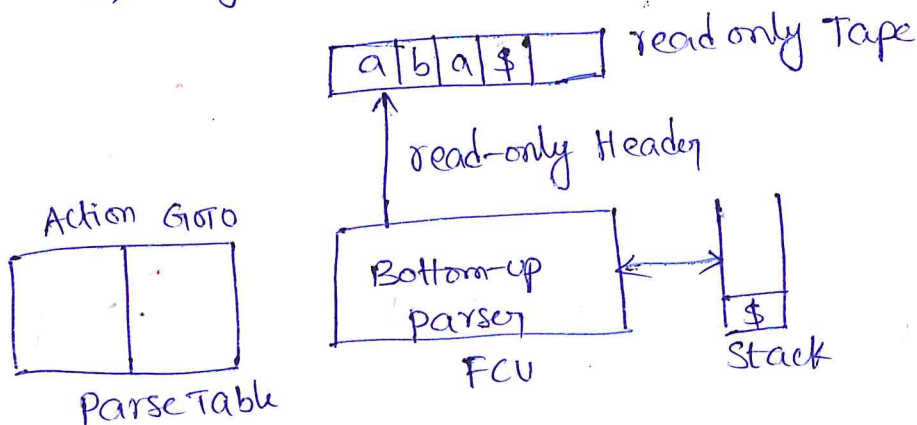
$$G \begin{cases} S \rightarrow aABe \\ A \rightarrow bc \\ B \rightarrow d \end{cases}$$



$$w = abcde$$



\hookrightarrow Shift-Reduce parsers

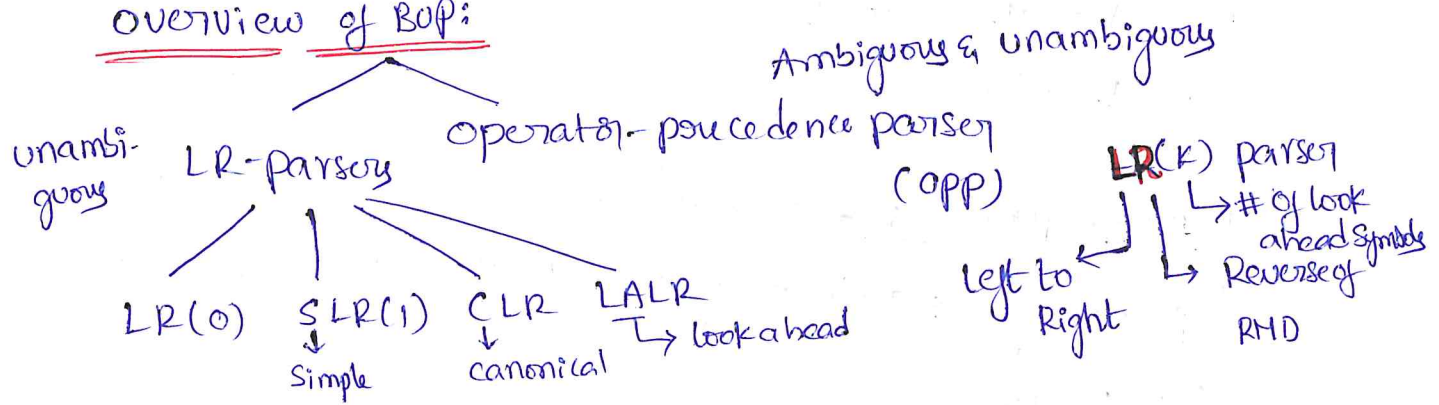


operations:

- Shift ✓
- Reduce ✓

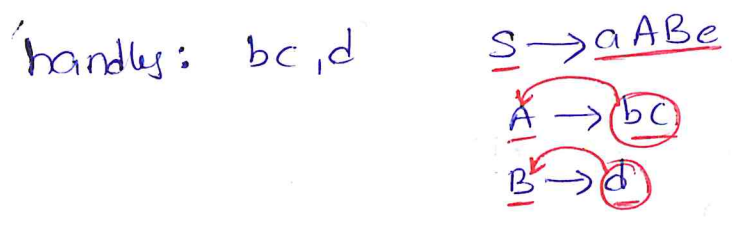
- ✓ Accept $\Rightarrow w \in L(G)$
- ✓ Error $\Rightarrow w \notin L(G)$

Overview of BOP:

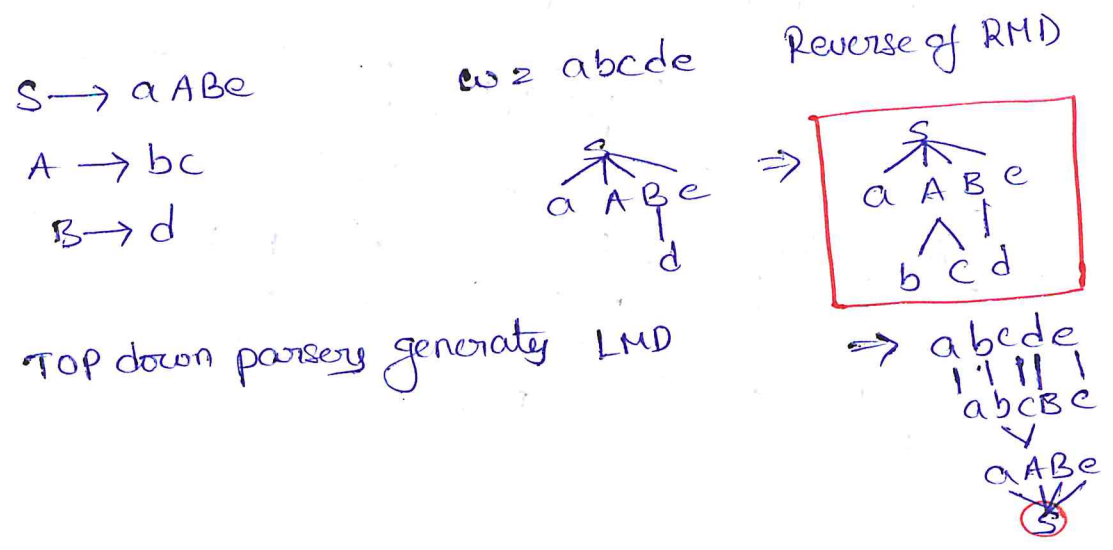


Handle & Handle-pairing:

\rightarrow part of the input string that matches the RHS of a production rule.



Replacing the Handle, with its LHS variable is known as Handle pairing.



Example of Shift & Reduce operations:

| | Stack | input | Action | Algorithm determining the action |
|--------------------|-------|--------|---------------------------|----------------------------------|
| $S \rightarrow AA$ | \$ | abab\$ | Shift | |
| $A \rightarrow aA$ | \$a | bab\$ | Shift | |
| $A \rightarrow b$ | \$ab | ab\$ | Reduce $A \rightarrow b$ | |
| | \$aA | ab\$ | Reduce $A \rightarrow aA$ | |
| | \$A | ab\$ | Shift | |
| | \$Aa | b\$ | Shift | |
| | \$Aab | \$ | Reduce $A \rightarrow b$ | |
| | \$AaA | \$ | Reduce $A \rightarrow aA$ | |
| | \$AA | \$ | Reduce $S \rightarrow AA$ | |
| | \$S | \$ | Accept | |

LR(0): Construct the parse table:

- ① Construct the Augmented Grammar
 - $G: A \rightarrow aA$
 - $A \rightarrow b$
 - ② Construct canonical collection of LR-items
 - $G': A' \rightarrow A$
 - $A \rightarrow aA$
 - $A \rightarrow b$
- LR-item: Item of the Compiler
- $A \rightarrow \cdot abc$

$$\left\{ \begin{array}{l} A \rightarrow a \cdot bc \\ A \rightarrow \cdot aA \\ A \rightarrow a \cdot A \end{array} \right.$$

Final-items:

$$\begin{array}{l} A \rightarrow aA \cdot \\ A \rightarrow ab \cdot \\ \boxed{A \rightarrow b \cdot} \end{array}$$

Canonical Collection:

$$C = \{ I_0, I_1, I_2, \dots, I_n \}$$

↑
Set of items

closure(I):

$$G': \left. \begin{array}{l} A' \rightarrow \cdot A \\ A \rightarrow \cdot aA \\ A \rightarrow \cdot b \end{array} \right\} \text{items}$$

closure($A' \rightarrow \cdot A$)

$$\{ \begin{array}{l} A' \rightarrow \cdot A \\ A \rightarrow \cdot aA \\ A \rightarrow \cdot b \end{array} \}$$

I_0

closure($A \rightarrow \alpha \cdot B \beta$)

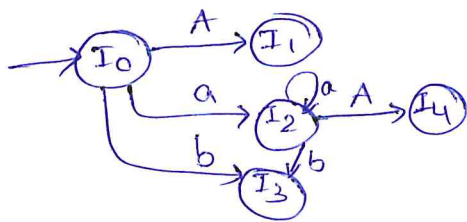
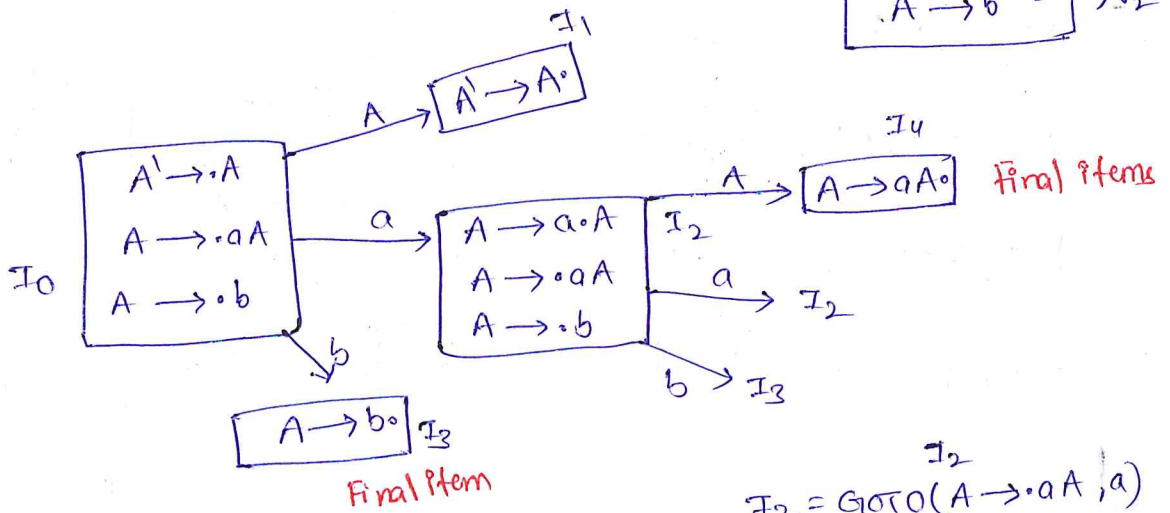
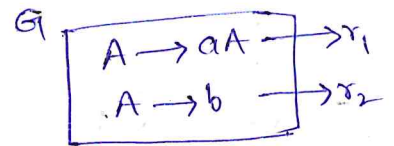
$$\downarrow$$

$$\begin{array}{l} B \rightarrow \cdot \gamma \\ \boxed{\begin{array}{l} A \rightarrow \alpha \cdot B \beta \\ B \rightarrow \cdot \gamma \end{array}} \end{array}$$

Eg 1: closure($A \rightarrow \cdot aA$) = $\{ A \rightarrow \cdot aA \}$

② GOTO(I, X): $Goto(\underbrace{A \rightarrow \alpha \cdot X \beta}_\text{item}, \underbrace{X}_\text{Symbol}) = \underbrace{(A \rightarrow \alpha X \cdot \beta)}_\text{item}$ (closure)

③ Construct the graph/DFA:



$$I_2 = \text{GOTO}(A \rightarrow \cdot aA, a)$$

$$= \text{closure}(A \rightarrow a \cdot A)$$

$$I_3 = \text{GOTO}(A \rightarrow \cdot b, b)$$

$$= \text{closure}(A \rightarrow b \cdot)$$

④ Construct parse-table:

| Terminals | Action | | | GOTO | |
|-----------|----------------|----------------|----------------|------|----------|
| | a | b | \$ | A | Variable |
| I_0 | s ₂ | s ₃ | | 1 | |
| I_1 | | | Accept | | |
| I_2 | s ₂ | s ₃ | | 4 | |
| I_3 | r ₂ | r ₂ | r ₂ | | |
| I_4 | r ₁ | r ₁ | r ₁ | | |

s: shift
r: Reduce

LR(0) parse table

⑤ Using the table to parse the input aab

| Stack | Input | Action |
|---------|-------|----------------|
| \$0 | aab\$ | s ₂ |
| \$0a2 | ab\$ | s ₂ |
| \$0a2a2 | b\$ | s ₃ |

(16)

| <u>stack</u> | <u>input</u> | <u>Action</u> |
|--------------|--------------|--------------------------|
| \$0a2a2b3 | \$ | r_2 $A \rightarrow b$ |
| \$0a2a2A4 | \$ | r_1 $A \rightarrow aA$ |
| \$0a2A4 | \$ | r_1 $A \rightarrow aA$ |
| \$0A1 | \$ | Accept |



LR(0) parser: M8re Examples:

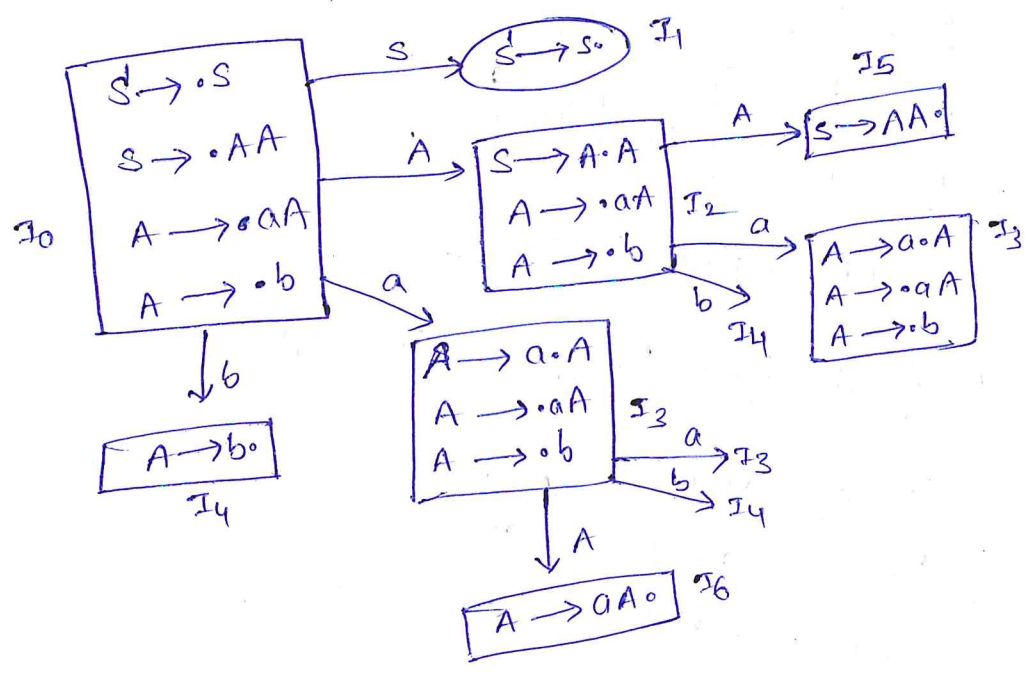
$w = abb$

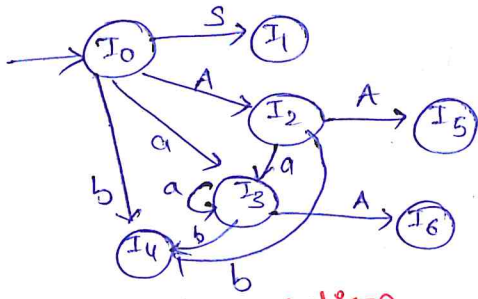
- $G_1: S \rightarrow AA \quad r_1$
 $A \rightarrow aA \quad r_2$
 $A \rightarrow b \quad r_3$

① Augmented Grammar

- $S' \rightarrow S$
 $S \rightarrow AA$
 $A \rightarrow aA$
 $A \rightarrow b$

② Find canonical collection of items





| | <u>Action</u> | | \$ | <u>GOTO</u> | |
|----------------|----------------|----------------|----------------|-------------|---|
| | a | b | | S | A |
| I ₀ | S ₃ | S ₄ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | S ₃ | S ₄ | | | 5 |
| I ₃ | S ₃ | S ₄ | | | 6 |
| I ₄ | r ₃ | r ₃ | r ₃ | | |
| I ₅ | r ₁ | r ₁ | r ₁ | | |
| I ₆ | r ₂ | r ₂ | r ₂ | | |

| <u>Stack</u> | <u>input</u> | <u>Action</u> |
|--------------|--------------|------------------------|
| \$0 | abb\$ | S ₃ |
| \$0a3 | bb\$ | S ₄ |
| \$0a3b4 | b\$ | r ₃ A → b |
| \$0a3A | b\$ | 6 |
| \$0a3A6 | b\$ | r ₂ A → aA |
| \$0A | b\$ | 2 |
| \$0A2 | b\$ | S ₄ |
| \$0A2b4 | \$ | r ₃ A → b |
| \$0A2A5 | \$ | r ₁ \$ → AA |
| \$0S | \$ | 1 |
| \$0S1 | \$ | <u>Accept</u> |

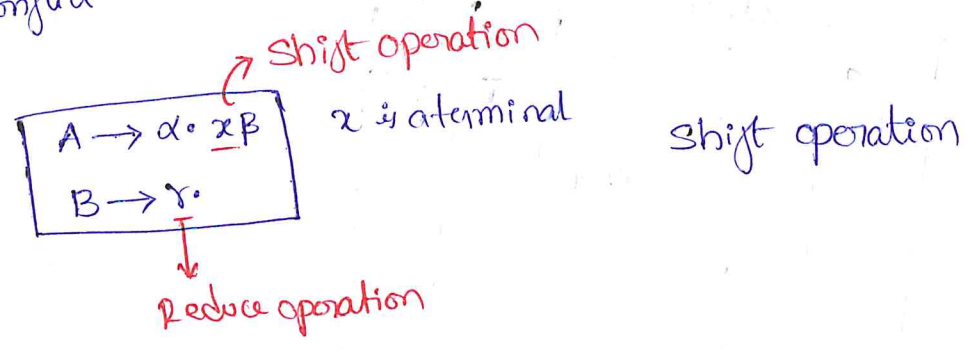
abb
w ∈ L(G)

Conflicts: LR(0)

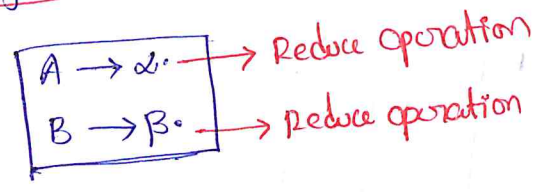
↳ if \exists multiple entries in a cell in the parse-table.

① SR Conflict

② RR Conflict

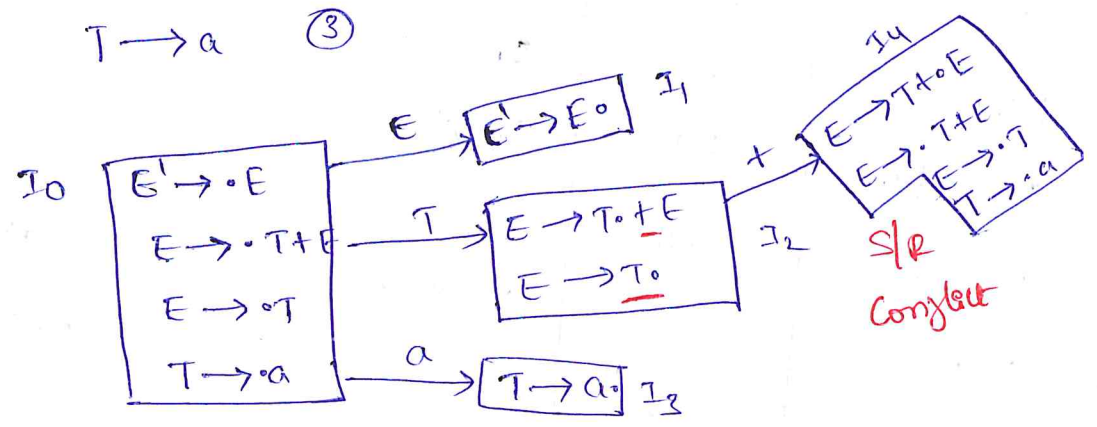


RR Conflict:



Eg 1:

- $E \rightarrow T + E$ ①
- $E \rightarrow T$ ②
- $T \rightarrow a$ ③



GOTO(I4, E)
 $E \rightarrow T + E .$ I5

GOTO(I4, T) I9

GOTO(I4, a) I2

| | + | a | \$ | E | T |
|----------------|------------------------------------|----------------|----------------|---|---|
| I ₀ | | S ₃ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | S ₄ / r ₂ | r ₂ | r ₂ | | |
| I ₃ | r ₃ | r ₃ | r ₃ | | |
| I ₄ | | S ₃ | | 5 | 2 |
| I ₅ | r ₁ | r ₁ | r ₁ | | |

SLR Conflict

NOT LR(0) Grammar

NOT LL(1) Grammar

$E \rightarrow T+E$
 $E \rightarrow T$

LR(0) Grammar

↳ if LR(0) parse table has no conflicts.

SLR(1) parser:

↳ Simple Extension to LR(0) parser.
 ↓
 parse-table (Mini)

G

$E \rightarrow T+E$ ①

$E \rightarrow T$ ②

$T \rightarrow a$ ③

Instead of placing the reduce operation under all the terminals, place them under follow (LHS) variables.

SLR(1) parse table for the Grammar

| | + | a | \$ | E | T |
|----------------|----------------|----------------|----------------|---|---|
| I ₀ | | S ₃ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | S ₄ | | r ₂ | | |
| I ₃ | r ₃ | | r ₃ | | |
| I ₄ | | S ₃ | | 5 | 2 |
| I ₅ | | | r ₁ | | |

$E \rightarrow T$

Follow(E) = { \$ }

$T \rightarrow a$

Follow(T) = { +, \$ }

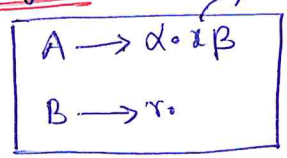
$E \rightarrow T+E$ r₁

$E \rightarrow T+E$

Follow(E) = { \$ }

SR & RR Conflicts in SLR(1) parse-table:

SR Conflict:



SR-conflict in LR(0)

SLR(1)

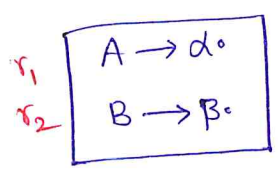
First(x) \cap Follow(B)

\downarrow

$x \cap \text{Follow}(B) \neq \emptyset$

SR Conflict in SLR(1)

RR Conflict:



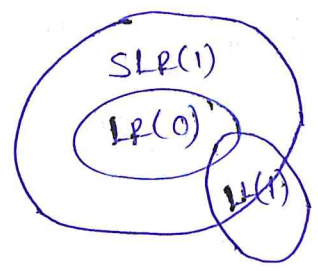
Follow(A) \cap Follow(B) $\neq \emptyset$

RR Conflict

SLR(1) Grammar:

\hookrightarrow if SLR(1) parse table has no conflicts.

Fewer entries than LR(0)

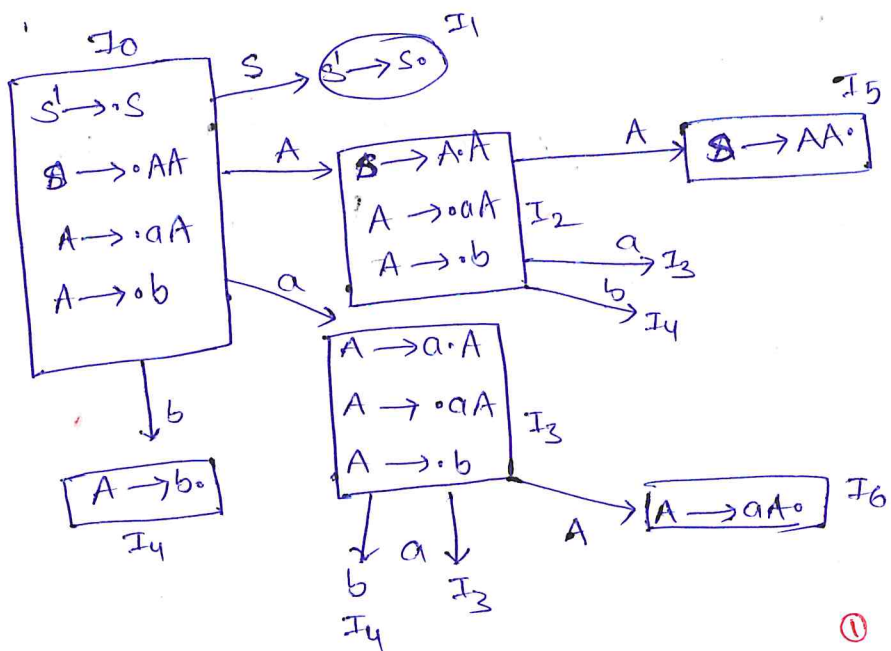


SLR(1) is more powerful than LR(0) parser.

LL(1) is not a proper subset of LR(0)

G1:

- $S' \rightarrow \cdot s$
- 1. $s \rightarrow \cdot AA$
- 2. $A \rightarrow \cdot aA$
- 3. $A \rightarrow \cdot b$



- ① $S \rightarrow AA$
- ② $A \rightarrow aA$
- ③ $A \rightarrow b$

Follow(A) = {a, b, \$}

Follow(S) = {\$}

| | a | b | \$ | S | A |
|----------------|----------------|----------------|----------------|----------------|---|
| I ₀ | s ₃ | s ₄ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | s ₃ | s ₄ | | | 5 |
| I ₃ | s ₃ | s ₄ | | | 6 |
| I ₄ | r ₃ | r ₃ | r ₃ | | |
| I ₅ | | | | r ₁ | |
| I ₆ | r ₂ | r ₂ | r ₂ | | |

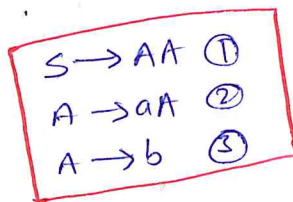
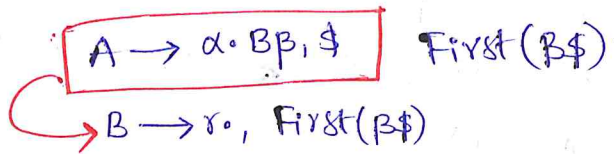
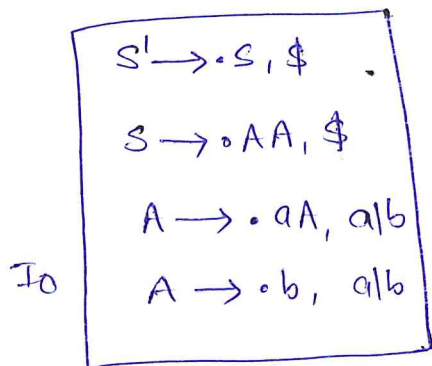
Both LR(0) and SLR(1)

No LR & RLR conflicts.

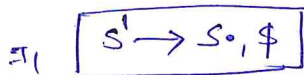
CLR(1) & LALR(1) :

LR(1) items : LR(0) items + lookahead

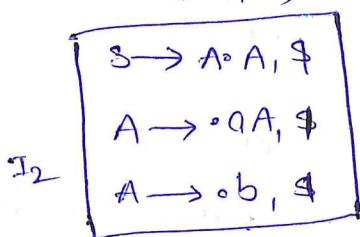
Eg: $S \rightarrow \dot{A}A$ \Rightarrow Augmented Grammar
 $A \rightarrow \dot{a}A$
 $A \rightarrow \dot{b}$
 $s' \rightarrow S$
 $S \rightarrow AA$
 $A \rightarrow aA$
 $A \rightarrow b$



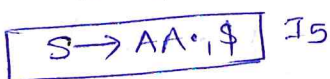
GOTO (I_0, S)



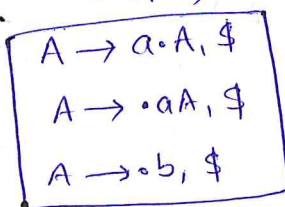
GOTO (I_0, A)



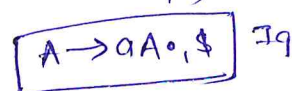
GOTO (I_2, A)



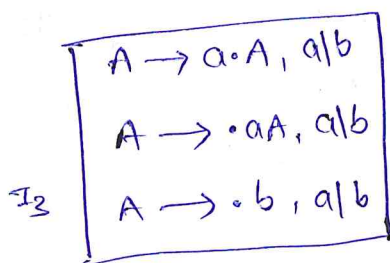
GOTO (I_2, a)



GOTO (I_6, A)



GOTO (I_0, a)



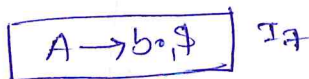
GOTO (I_6, a)

I_6

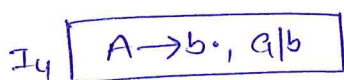
GOTO (I_6, b)

I_7

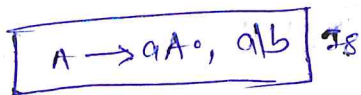
GOTO (I_2, b)



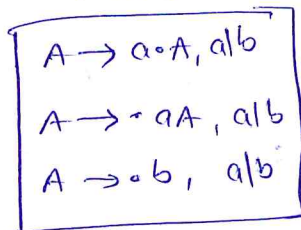
GOTO (I_0, b)



GOTO (I_3, A)

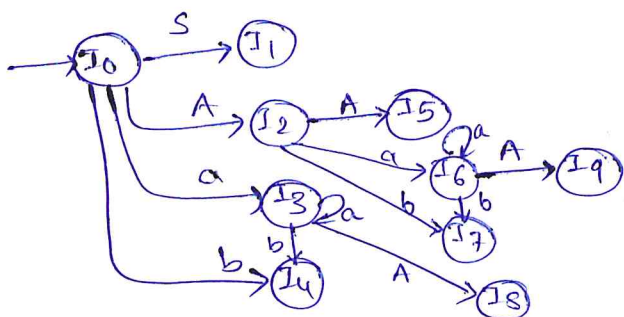


GOTO (I_3, a)



GOTO (I_3, b)

I_4



| | a | b | \$ | S | A |
|----------------|----------------|----------------|----------------|---|---|
| I ₀ | S ₃ | S ₄ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | S ₆ | S ₇ | | | 5 |
| I ₃ | S ₃ | S ₄ | | | 8 |
| I ₄ | r ₃ | r ₃ | | | |
| I ₅ | | | r ₁ | | |
| I ₆ | S ₆ | S ₇ | | | 9 |
| I ₇ | | | r ₃ | | |
| I ₈ | r ₂ | r ₂ | | | |
| I ₉ | | | r ₂ | | |

Number of states in CLR(1) \geq SLR(1)

CLR(1) parser



placing r_i in fewer cells than SLR(1)



lesser chance of conflict

LR(0): Fullrow

SLR(1): Follow of LHS

CLR(1): Look ahead

LALR(1) parser:

3&6, 4&7, 8&9

I₃₆ I₄₇ I₈₉

| | a | b | \$ | S | A |
|-----------------|-----------------|-----------------|----------------|---|----|
| I ₀ | S ₃₆ | S ₄₇ | | 1 | 2 |
| I ₁ | | | Accept | | |
| I ₂ | S ₃₆ | S ₄₇ | | | 5 |
| I ₃₆ | S ₃₆ | S ₄₇ | | | 89 |
| I ₄₇ | r ₃ | r ₃ | r ₂ | | |
| I ₅ | | | r ₁ | | |
| I ₈₉ | r ₂ | r ₂ | r ₂ | | |

Merge the states

NO. of states in LR(0)

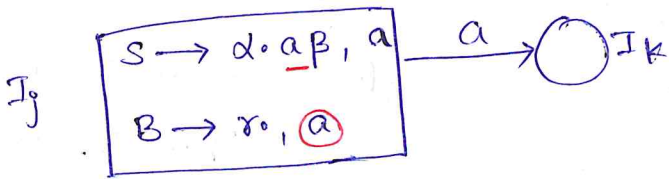
= SLR(1) = LALR(1)

≤ CLR(1)

CLR(1) and LALR(1) Grammar and Example:
 → LR(1)

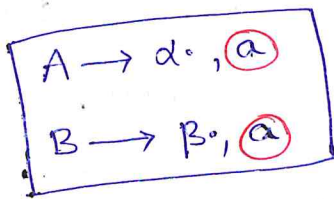
SLR(1), LR(0)

Conflicts : S/R & R/R



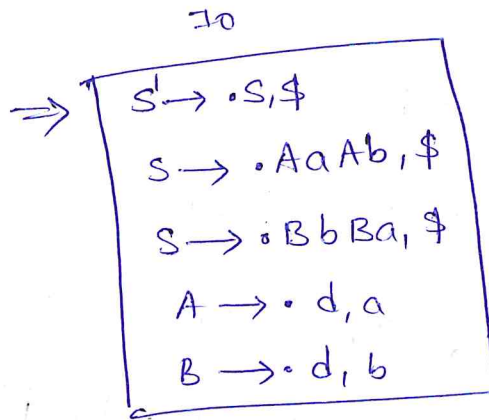
S/R Conflict in CLR(1)

| | |
|-------|-----------|
| | a |
| I_0 | S_k |
| I_3 | r_1/r_2 |

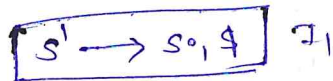


R/R Conflict in CLR(1)

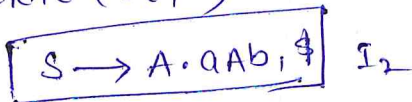
- Gr:
- ① $S \rightarrow AaAb$
 - ② $S \rightarrow BbBa$
 - ③ $A \rightarrow d$
 - ④ $B \rightarrow d$



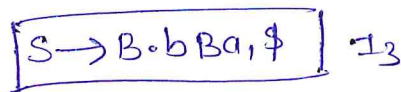
GOTO(I_0, S)



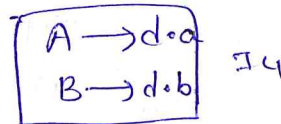
GOTO(I_0, A)



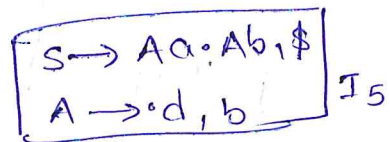
GOTO(I_0, B)



GOTO(I_0, d)



GOTO(I_2, a)



GOTO(I₃, b)

$S \rightarrow Bb \cdot Ba, \$$
 $B \rightarrow \cdot d, a$ I₆

GOTO(I₅, A)

$S \rightarrow AaA \cdot b, \$$ I₇

GOTO(I₇, b)

$S \rightarrow AaAb \cdot, \$$ I₁₁

GOTO(I₅, d)

③ $A \rightarrow d \cdot, b$ I₈

GOTO(I₉, a)

$S \rightarrow BbBA \cdot, \$$ I₁₂

GOTO(I₆, B)

$S \rightarrow BbB \cdot a, \$$ I₉

GOTO(I₆, d)

④ $B \rightarrow d \cdot, a$ I₁₀

No SLR and RLR Conflicts. CLR(1) Grammar.

| | a | b | d | \$ | S | A | B |
|-----------------|-----------------|---------------------------|-----------------|----------------|---|---|---|
| I ₀ | | S ₄ | | | 1 | 2 | 3 |
| I ₁ | | | | Accept | | | |
| I ₂ | S ₅ | | | | | | |
| I ₃ | | S ₆ | | | | | |
| I ₄ | r ₃ | r ₄ | | | | | |
| I ₅ | | | S ₈ | | | | 7 |
| I ₆ | | S₁₀ | S ₁₀ | | | | 9 |
| I ₇ | | S ₁₁ | | | | | |
| I ₈ | | r ₃ | | | | | |
| I ₉ | S ₁₂ | | | | | | |
| I ₁₀ | r ₄ | | | | | | |
| I ₁₁ | | | | r ₁ | | | |
| I ₁₂ | | | | r ₂ | | | |

No conflicts
 CLR(1) Grammar

LALR(1) Grammar & Conflicts:



Combining states in CLR(1) parse table.

→ If CLR(1) parse table has no LR conflicts then no LR conflicts in LALR(1).

→ If CLR(1) parse table has no RR conflicts, then there may/may not exist RR conflicts in LALR(1).

Eg 1:

1 $S \rightarrow Aa$

2 $S \rightarrow bAc$

3 $S \rightarrow Bc$

4 $S \rightarrow bBa$

5 $A \rightarrow d$

6 $B \rightarrow d$

$S' \rightarrow S$

$S \rightarrow Aa$

$S \rightarrow bAc$

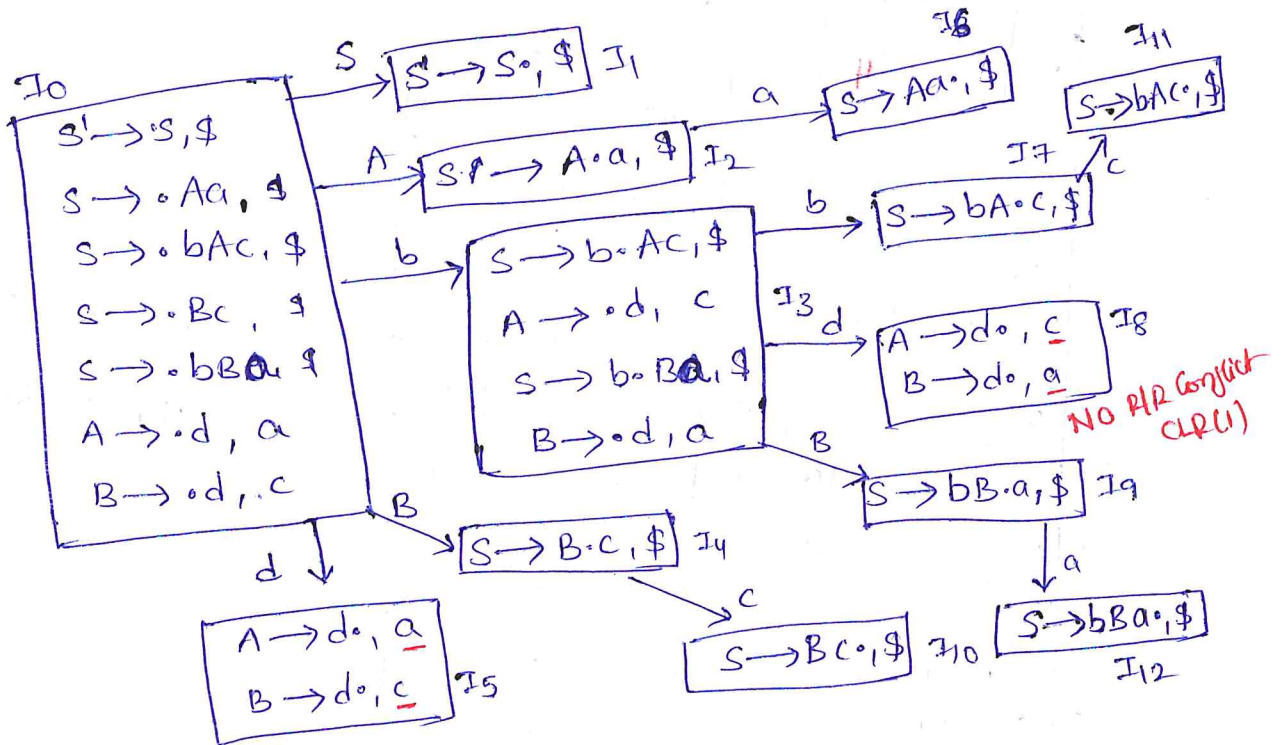
$S \rightarrow Bc$

$S \rightarrow bBa$

$A \rightarrow d$

$B \rightarrow d$

CLR(1) but not LALR(1)



No Conflict in CLR(1)

No Conflicts in CLR(1)

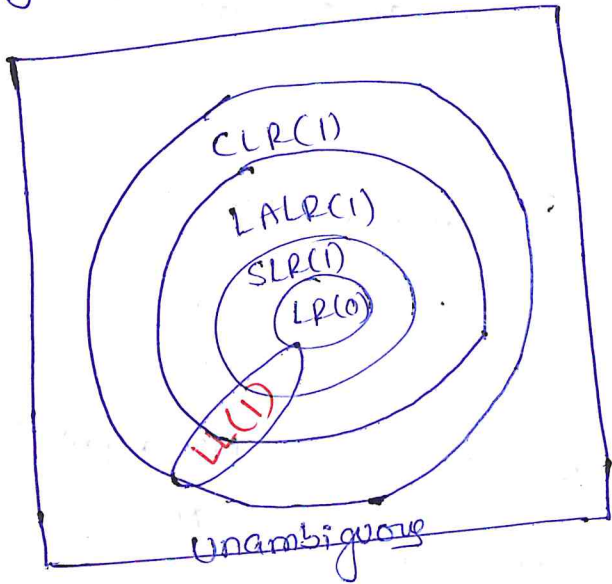
I₅₈

| | | | | | | |
|---|---|---|---|---|----------|----------|
| 5 | A | → | d | . | <u>a</u> | <u>c</u> |
| 6 | B | → | d | . | <u>a</u> | <u>c</u> |

R/R Conflict in LALR(1)

| | | |
|-----------------|-------|-------|
| | a | c |
| I ₅₈ | rs/r6 | rs/r6 |

Every LALR(1) is CLR(1), but not vice versa.



CLR(1) parser is more powerful.

LALR(1) parser → practice
↑
fewer states

operator precedence parser:

- ↳ BOP, ambiguous & unambiguous
- ↳ cannot work for all CFG
- ↳ +, -, *, /, %

operator Grammar:

- ↳ CFG { no ε-transitions
- { No AB on RHS
- ↳ Adjacent variables

AaB all terminals
 ↓
 operator

Eg: ① $S \rightarrow AB$

$A \rightarrow a$
 $B \rightarrow b$

NOT opG
operator Grammar

② $S \rightarrow AaB$

$A \rightarrow aA|b$
 $B \rightarrow bB|a$

No ϵ -prod
No Adjacent Variables

③ $S \rightarrow AaB$

$A \rightarrow aA|b$
 $B \rightarrow bB|\epsilon$

Not an operator Grammar

④ $E \rightarrow EAE|id$
 $A \rightarrow +|-|*$

} Adjacent variables
→ not -opG

⇓

$E \rightarrow E+E|E-E|E \times E|id$

No adjacent variables
No ϵ -productions

operator Grammar

Ambiguous Grammar

→ $G: E \rightarrow E+E$

$E \rightarrow E * E$
 $E \rightarrow id$

Ambiguous op. Gr

$w = id + id * id \$$

< shift

> pop, Reduce

stack
\$

input
id + id * id \$

Action
push

\$ id

+ id * id \$

pop +
Reduce

Empty Error

\$ *

+ id * id \$

push

\$ +

id * id \$

push

\$ + id

* id \$

pop, Reduce

| | | | | |
|----|----|---|---|----|
| | id | + | * | \$ |
| id | E | > | > | > |
| + | < | > | < | > |
| * | < | > | > | > |
| \$ | < | < | < | E |

n x n

precedence Table

$2 + 3 * 5 = 2 + 15 = 17$

$2 + 3 + 4 \Rightarrow 5 + 4 \Rightarrow 9$

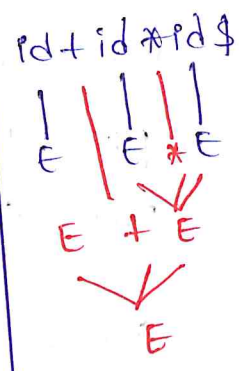
$3 + 2 * 5$

↓ LA

id + id * id \$

operator precedence Grammar

| Stack | input | Action |
|-----------|---------|--------------|
| \$ + | < id \$ | push |
| \$ + * | < id \$ | push |
| \$ + * id | > \$ | pop & Reduce |
| \$ + * | > \$ | pop & Reduce |
| \$ + | > \$ | pop & Reduce |
| \$ | \$ | Accept |

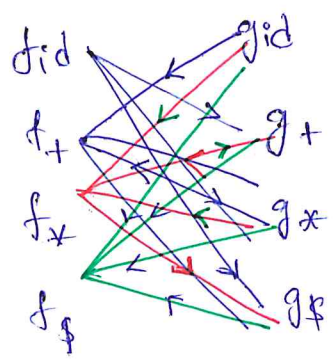


$O(n^2)$ to $O(2n)$ table:

| | | | | |
|----|----|---|---|----|
| | id | + | * | \$ |
| id | | > | > | > |
| + | < | > | < | > |
| * | < | > | > | > |
| \$ | < | < | < | |

Rel Table

① Functional Table No-cycles



② $f_{id} \rightarrow g_x \rightarrow f^+ \rightarrow g^+ \rightarrow f_\$$

$g_{id} \rightarrow f_x \rightarrow g_x \rightarrow d^+ \rightarrow g^+ \rightarrow f_\$$

Compute Largest path

③

| | | | | |
|---|---|---|----|----|
| | + | * | id | \$ |
| f | 2 | 4 | 4 | 0 |
| g | 1 | 3 | 5 | 0 |

Functional Table

$O(2n)$

Compare (+, +)

Compare (id, id)

f id g +
4 > 1

f + g *
2 < 3

f id → g id
Error

Compare (id, +)

Functional table captures all the information from the precedence table, but they will not get the information in empty cell in precedence table.

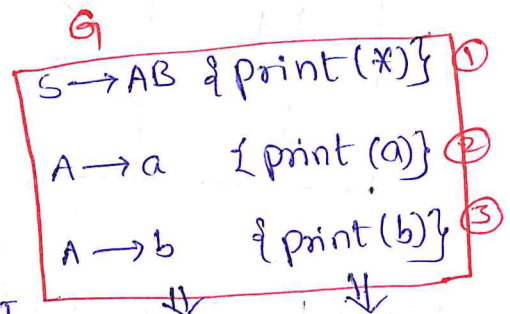
Error detection will be done efficiently in precedence Table.

Syntax Directed Translation: (SDT)

Src. Code

↳ Lexical Analysis

→ Syntax Analysis



↓ PT

Semantic Analysis CFG + Actions = SDT

↓

TCG

↓

code optimization

↓

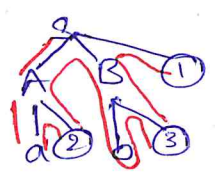
Code Generation.

CFG + Semantic Actions / Rules

Type checking

let $w = ab$

output: $ab*$



parse tree

TDP

Topdown

$L \rightarrow R$

input = ab

output : ab* postfix Expression.

what can be done using SDT?

Dragon book

→ Type-checking

→ Algebraic Expression Evaluation

→ Syntax-tree

→ Verify variable declaration

Annotated Parse Tree: (Decorated parse Tree)

G: $E \rightarrow E + E$ { $E.val = E.val + E.val$ } ①

$E \rightarrow E * E$ { $E.val = E.val * E.val$ } ②

$E \rightarrow id$ { $E.val = id.val$ } ③

CFG

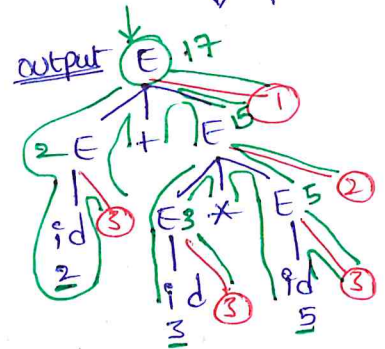
action

w = 2 + 3 * 5

↓
lex. Analysis (Symbol Table)

↓
w = id + id * id

↓ parser(TOP)



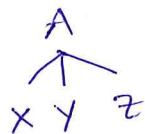
Top-down
L → R

SDT:

$E.val$
 $id.val$ } Evaluate algebraic Expression

Types of attribute: Synthesized & Inherited

① $A \rightarrow XYZ$



S is an attribute corresponding to A

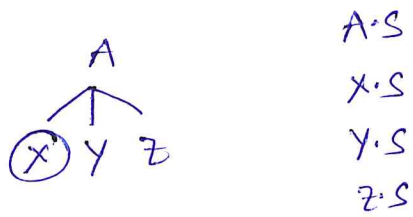
children only

$$A.S = f(X.S, Y.S, Z.S)$$

Attribute S is known as Synthesized attribute.

Inherited Attribute:

$$A \rightarrow XYZ$$



$$X.S = f(A.S, Y.S, Z.S)$$

parent and/or siblings.

Attribute S: Inherited Attribute

Constructing SDT with example:

① Construct SDT for expression Evaluation

$$E \rightarrow E + T \quad \{ E.val = E.val + T.val \} \textcircled{1}$$

$$E \rightarrow T \quad \{ E.val = T.val \} \textcircled{2}$$

$$T \rightarrow T * F \quad \{ T.val = T.val * F.val \} \textcircled{3}$$

$$T \rightarrow F \quad \{ T.val = F.val \} \textcircled{4}$$

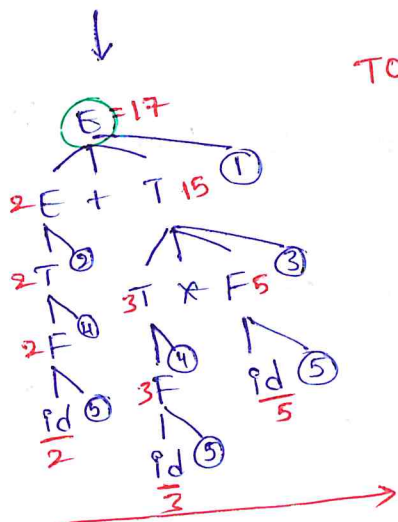
$$F \rightarrow id \quad \{ F.val = id.val \} \textcircled{5}$$

← Actions

$$W = 2 + 3 * 5$$

$$\downarrow LA + S \cdot T$$

$$id + id * id$$



Topdown
L → R

$$\text{input} = 2 + 3 * 5$$

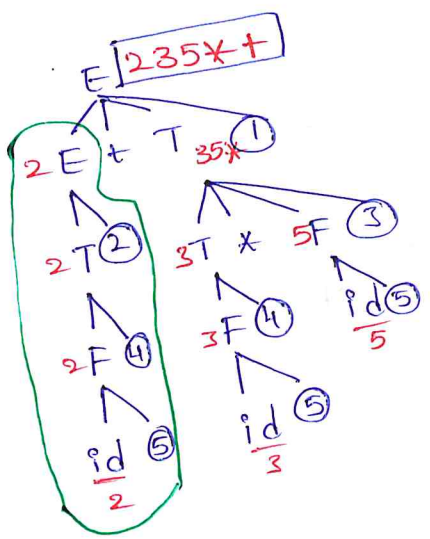
$$\text{output} = \underline{\underline{17}}$$

② SDT for infix to postfix Conversion:

input: 2+3*5
output: 235*

- $E \rightarrow E + T$ { $E.val = E.val T.val +$ }
 - $E \rightarrow T$ { $E.val = T.val$ }
 - $T \rightarrow T * F$ { $T.val = T.val F.val *$ }
 - $T \rightarrow F$ { $T.val = F.val$ }
 - $F \rightarrow id$ { $F.val = id.val$ }
- CFG Action

2+3*5
↓ LA+ST
id+id*id



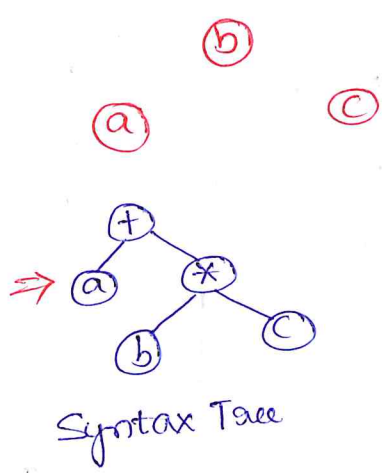
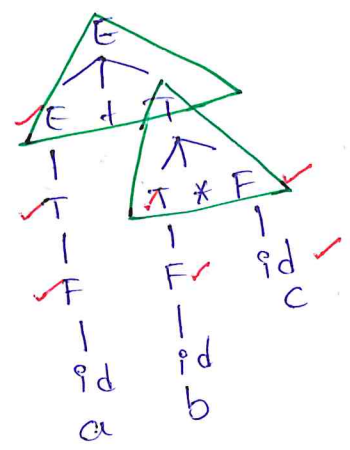
Top down
L → R
parsing: stack

③ SDT for Type-checking:

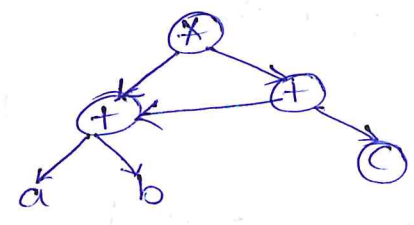
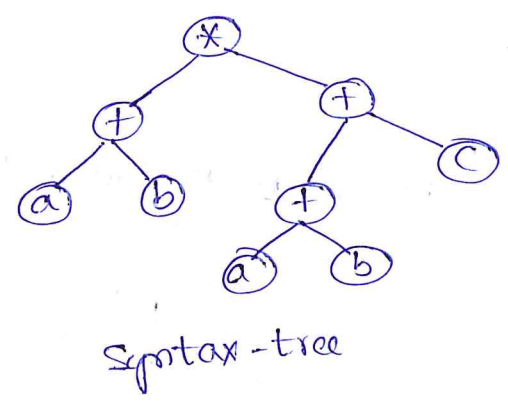
- $E \rightarrow E + T$ { if ($E.type == T.type$) && ($T.type == int$) then $E.type = int$ else Error }
- $E \rightarrow T$ { $E.type = T.type$ }
- ~~$T \rightarrow F$~~
- $T \rightarrow T * F$ { if ($T.type == F.type$) && ($F.type == int$) then $T.type = int$ else Error }
- $T \rightarrow F$ { $T.type = F.type$ }
- $F \rightarrow id$ { $F.type = id.type$ }

parse-tree: Terminal & non-terminals

In a Syntax-tree all ~~tree~~ are terminals, nodes



⑥ SDT for expression-eval DAGs:



Avoid duplicate computation

Types of SDT:

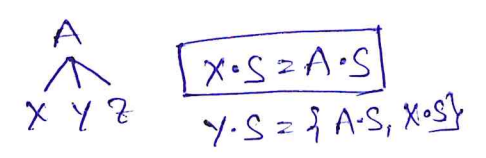
S-attributed SDT

① only uses synthesized attributes

L-attributed SDT

① uses both synthesized & inherited attributes

are restricted to using attributes from parent & left sibling only.

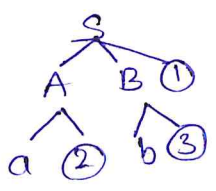


S-Attributed

L-Attributed

- ② actions are placed @ the end of RHS

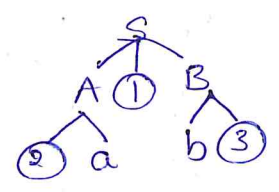
$S \rightarrow AB \{ \text{print } x \}$ ①
 $A \rightarrow a \{ \text{print } a \}$ ②
 $B \rightarrow b \{ \text{print } b \}$ ③



- ③ Attributes can be evaluated using Topdown or bottomup methods

- ② Actions can be placed anywhere on RHS.

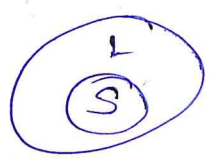
$S \rightarrow A \textcircled{1} B$
 $A \rightarrow \textcircled{2} a$
 $B \rightarrow b \textcircled{3}$



w=ab

- ③ Depth First L → R

Note: Every S-Attributed SDT is an L-Attributed SDT.



L-Attributed SDT:

S-Attributed SDT

$S \rightarrow A \textcircled{1} B$ → $\left\{ \begin{array}{l} S \rightarrow ARB \\ R \rightarrow \epsilon \end{array} \right\}$
 $A \rightarrow \textcircled{2} a$ → $\left\{ \begin{array}{l} A \rightarrow xa \\ x \rightarrow \epsilon \end{array} \right\}$
 $B \rightarrow b \textcircled{3}$

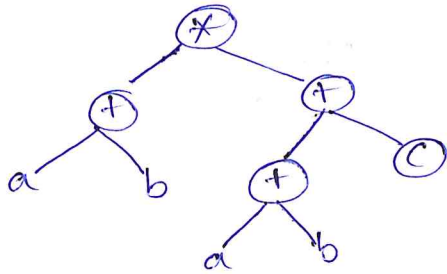
$B \rightarrow b \textcircled{3}$

ICG + 3-Address codes: Platform Independent

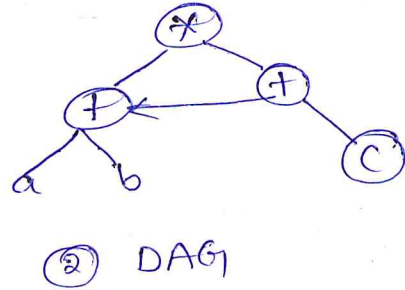
Improve code-generation efficiency.

$$(a+b) * (a+b+c)$$

Syntax Trees (Abstract Syntax Tree)



① Syntax Tree



② DAG

③ postfix: $ab+ ab+c+ *$

↳ one way of representing intermediate code.

④ Three-Address Code:

$$t_1 = a+b$$

$$t_2 = a+b$$

$$t_3 = t_2+c$$

$$t_4 = t_1 * t_3$$

3-address code.

In 3-address code atmost three addressees are there.

Representation of 3-address code:

$x = a + b * c$

$t_1 = b * c$

$t_2 = a + t_1$

$x = t_2$

① Quadruples

| | | | |
|-------|-----|-------|-----|
| t_1 | b | c | $*$ |
| t_2 | a | t_1 | $+$ |
| x | | t_2 | $=$ |

② Triply

① $bc *$

② $a① +$

③ $x② =$

No Temp
Var

③ Indirect Triply

(101) $bc *$

(102) $a(101) +$

(103) $x(102) =$

pointers
reuse
Computatory

Types of 3-addr codes:

① $x = y \text{ op } z$ ✓

② $x = \text{op } y$ ✓

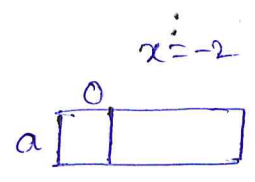
③ $x = y$ ✓

④ $x = *y$ ✓

⑤ $x = \&y$ ✓

⑥ $x = a[i]$ ✓

⑦ $a[i] = x$ ✓



⑧ GOTO L ✓ unconditional jump

⑨ if (expr) GOTO L ✓

⑩ $y = f(x_1, x_2, \dots, x_n)$ x
more than 3-addresses

① $x = a - b - c + d / e$

$t_1 = a - b$

$t_2 = t_1 - c$

$t_3 = d / e$

$t_4 = t_2 + t_3$

$x = t_4$

②

int x=1;

int y=2;

int z=x+y;

print(z);

x=1

y=2

z=x+y

goto L

L: print(z)

③

if (x > y)

print("TOC")

else

print("CO")

① if (x > y) GOTO L5 Cond-Jump

② ~~goto L2~~

③ L1: print("CO")

④ goto ⑥

→ ⑤ L2: print("TOC")

⑥ END/EXIT

④

for (int i=1; i<=10; i++)

{

int x = 2 * i;

print(x);

}

- ① $i = 1$
- ② $\text{if } (i > 10) \text{ goto } 9$: Conditional jump
- ③ $x = 2 * i$
- ④ $\text{Goto } L$: Unconditional
- ⑤ $L: \text{print}(x)$
- ⑥ } $t = i + 1$
- ⑦ } $i = t$
- ⑧ $\text{goto } ②$
- ⑨ END/EXIT

